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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

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MARCH 1916

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THE PHYSICAL JOURNAL

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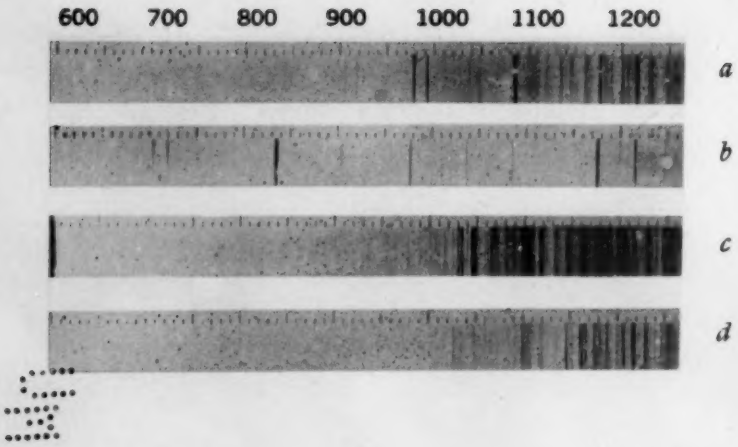
PLATE III.



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THE EXTENSION OF THE SPECTRUM BEYOND THE SCHUMANN REGION

By THEODORE LYMAN

I have recently published a brief statement of the result of my investigations on the extension of the spectrum.[1] Since the appearance of that article my researches in this field have been directed toward clearing up certain doubtful points. Although these experiments have not resulted in pushing the spectrum beyond the limit already announced, and although the investigation is by no means at an end, it now seems time to make a fuller statement than was possible in a preliminary notice. Such a statement is all the more desirable since an understanding of the difficulties which have been encountered is necessary if one is to estimate the probability of future progress. The first pages of this article, therefore, will be found to contain substantially the same material as the earlier notice, while the later pages are devoted to a rather detailed description of the experimental procedure.

The violet limit of the spectrum determined by direct eye-observation lies in the neighborhood of λ 4000; with a glass prism and lenses, the spectrum may be followed photographically to λ 3000 or thereabout; with a quartz system or with a reflecting grating, the limit may be pushed to λ 1850. Victor Schumann

[1] *Proceedings of the National Academy of Sciences*, 1, 368, 1915; *Nature*, 95, 343, 1915.

showed that the absorption of the air and of the gelatine of the photographic plate were responsible for the abrupt termination of the ultra-violet spectrum near λ 1850. By employing a vacuum spectroscope and a special photographic plate, the emulsion of which was very poor in gelatine, he was able to push his observations to λ 1230. At this point he was stopped by the opacity of the fluorite of which his lenses were made. I have employed a concave diffraction grating arranged in such a manner that the light-path from the source to the photographic plate is wholly in gas. Briefly, the containing apparatus may be described as a brass tube about 11 cm in diameter and rather over a meter long.[1] This tube is closed at each end by brass plates ground to fit suitable flanges. In my most recent experiments the light is generated electrically in a discharge tube of quartz provided with tungsten electrodes. This discharge tube fits air-tight on one of the two brass plates just mentioned; light from it, having passed through a slit, traverses the length of the apparatus and falls upon the diffraction grating by which, having been analyzed into its component colors, it is brought to focus on a special photographic plate placed in close proximity to the slit.

As the discharge tube is in no way separated from the body of the spectroscope, it is obviously necessary to choose for the experiment some gas which will not only yield radiations in the region under investigation but which will be transparent to these radiations. My earlier experiments were conducted with hydrogen, since it had shown the necessary characteristics in that part of the spectrum investigated by Schumann. With this gas at a pressure of 2 or 3 mm and by employing a strong disruptive discharge, in May 1914, I was able to extend the spectrum[2] to λ 900.

A rather tedious investigation having convinced me that nothing more was to be expected from the use of hydrogen, at least in this neighborhood, I turned my attention to helium, being guided by some of my earlier experiments which had proved that this gas possesses the necessary transparency.[3] At the same time I made some improvements in my apparatus which, though they left its

[1] Lyman, *Spectroscopy of the Extreme Ultra-Violet* (Longmans, Green, & Co.), p. 34.

[2] *Ibid.*, p. 78; *Nature*, 93, 241, 1914.

[3] *Ibid.*, p. 69.

general form unchanged, resulted in making it far more air-tight than ever before. The success of this improvement may be judged by the fact that I have been able recently to leave the spectroscope for over fourteen hours at a pressure of about 3 mm without being able to detect any leak, either by a McLeod gauge reading to 0.007 mm or by the appearance of impurities in the spectrum of the gaseous content. I also took particular pains to purify the helium which I employed.

I have been rewarded for my trouble by a very considerable extension of the spectrum, for with helium free from nitrogen, at a pressure of 2 or 3 mm, by the use of the disruptive discharge, and with an exposure of about ten minutes, I have repeatedly observed a number of new lines the most refrangible of which has a wave-length of 600 A. All this has been accomplished with a grating ruled on speculum and with photographic plates prepared exactly as recommended by Schumann. The line λ 600 represents an extension beyond Schumann's limit greater than that which Schumann himself achieved beyond the region of λ 1850.

The relations of the spectra of hydrogen and helium have recently come into prominence through the theoretical researches of Bohr, Nicholson, and others. It so happens that the region on the more refrangible side of λ 1250 offers an important ground for the study of these relations. In order that the conclusions may be of value, however, it is necessary that the gases under observation should be free from impurities. In the best form of closed discharge tubes the difficulties, arising from the contamination of the gas by the material of the tube or by the electrodes, are very considerable, especially when a disruptive discharge is used. In my apparatus, where the brass spectroscope and quartz discharge tube communicate directly, these difficulties are considerably increased. I trust in time to overcome them, but for the present I must confine myself to the following statements.

Prominent in the spectrum of hydrogen is the line at λ 1216 which forms the first member of a series predicted, on theoretical grounds, by Ritz.[1] I have also found the two next members near λ 1026 and λ 972. With pure hydrogen, λ 972 is not visible

[1] W. Ritz, *Gesammelte Werke*, p. 105.

and λ 1026 is best seen when a disruptive discharge is used, but singularly enough, with helium containing a trace of hydrogen, both lines occur quite strongly with a simple alternating current from a 60-cycle transformer. This may be an illustration of the curious behavior of helium, for an atmosphere of this gas seems to facilitate the production of the spectra of other gaseous substances which occur in it as impurities.

In addition to these lines predicted by Ritz there are a number of others of equal prominence which always appear with a disruptive discharge in hydrogen. Two of these are of special interest, for their frequencies may be calculated rather exactly from the formula:

$$v = N\left\{1 - \frac{1}{(n+0.5)^2}\right\};$$

they persistently occur at λ 1086 and λ 992. Though the matter is perhaps not entirely settled, all the evidence seems to indicate that the two lines in question are due to an impurity.

The radiations from atmospheric nitrogen and from argon have also been examined. With both substances the disruptive discharge is necessary to produce the best results. The spectrum of the former gas with this type of excitation possesses some strong lines between λ 1050 and λ 1250; the spectrum terminates, however, near λ 975. With a simple 60-cycle alternating current, argon shows a spectrum terminating near λ 915. The extent of its spectrum is thus superior to that of hydrogen or of helium when subjected to similar electric conditions. With a disruptive discharge argon gives a strong spectrum of many lines terminating abruptly near λ 800.

As far as I am able to observe, with one exception all the lines seen with helium when excited by a non-disruptive discharge are also found in hydrogen; a disruptive discharge is necessary for the production of the new lines of extremely short wave-length. The intensity of these lines seems to depend in a critical way on the electrical conditions in the exciting circuit, for it is only when the pressure in the discharge tube, the external spark-gap, and the capacity are all nicely adjusted that the extreme lines appear to the best advantage. The nature of the adjustments suggests that

the radiations of the shortest wave-lengths require for their production that the atoms or molecules within the gas possess a very high velocity at the moment of collision. Some of the points in the preceding paragraphs are illustrated in Plate III.

Spectrum *a* is the spectrum obtained from hydrogen when a spark-gap of about 3 mm is placed in series with the tube and when a capacity of 0.014 microfarad is shunted across the terminals of the transformer. Pressure about 1 mm. Time of exposure about 12 minutes.

Spectrum *b* is the spectrum of helium containing hydrogen under the same electrical conditions as for spectrum *a*. Pressure 1.7 mm. Time of exposure 8 minutes.

Spectrum *c* is the spectrum of hydrogen excited by a 60-cycle alternating current, there is no capacity in the system beyond that of the leads; current about 15 milliamperes. Pressure 2–3 mm. Time of exposure one hour.

Spectrum *d* is the spectrum of helium under the same electrical conditions as for spectrum *c*. Pressure 2.2 mm. Time of exposure 30 minutes.

The extreme lines in the helium spectrum are probably not visible in the reproduction. The scales are intended only to give the position of the lines approximately. I crave the reader's indulgence for the blemishes which disfigure the spectra; they are almost unavoidable under the conditions of the experiment.

The wave-length of the X-rays, as determined by the Braggs, is of the order of one angstrom. There is therefore a gap of some six hundred units between the region of the ordinary Roentgen rays and the limit which I have reached. Several attempts have been made recently to produce less refrangible X-rays, but there is no satisfactory way of estimating their wave-length unless perhaps one relies on the Planck-Einstein formula $Ve=h\nu$. Taking V as the potential drop which gives the cathode particle the velocity necessary to produce a very soft X-ray, e as the charge on the particle, and giving to h the usual value ascribed to Planck's constant, the experiments of Dember[1] were made to yield a wave-length for the X-ray longer than that which I have determined.

[1] Lyman, *Spectroscopy*, p. 108.

Such speculations, though they are very interesting, cannot be given any great weight. There is still a considerable region between λ 600 and wave-length 1 angstrom which must be experimentally explored. My present limit is probably due rather to difficulties of technique than to any fundamental fault in method. Now that we know that a Schumann plate can be used and that the speculum grating is efficient, I see no insuperable difficulty to a still further extension by purely spectroscopic means.

The following pages contain a detailed description of my experimental procedure. A table of the wave-lengths of the new lines is given near the end of the paper.

The vacuum grating spectroscope with which this research has been carried out has been already fully described.[1] The changes on which the success of the present research largely rest are simple in character but of fundamental importance. Their object was to improve the air-tightness of the containing vessel and to guard the purity of the gas content. To this end, the plate which closed the working end of the container was of the simpler form used in my earlier work. It was of brass 1.3 cm thick ground to fit the flange which it was destined to cover; of its two windows, one was permanently provided with a quartz disk through which comparison spectra were obtained, the other was fitted with a cup into which a brass cone, destined to carry the discharge tube, was ground. The plate was attached to the flange of the spectroscope in the following way: A string of pure beeswax, some 2 mm in diameter, was bent into a circle of the required size; the flange of the receiver was then slightly warmed and the wax was lightly pressed against it; a photographic plate having been put in place, the face plate, also slightly warmed, was pressed against the wax. On the receiver being exhausted, the pressure of the air forced the face plate against the flange and thus squeezed the wax into a thin gasket. This device was suggested to me by Professor R. J. Strutt; by its means it has been possible to reduce the leak in the apparatus to a value far smaller than I have been able to attain by any other method. An example of its efficiency has already been given. As a high degree of air-tightness is fundamental to

[1] Lyman, *op. cit.*, p. 34.

the success of the whole research, the importance of the wax gasket is evident. It is obvious that the nature of the arrangement just described necessitates the removal of the face plate whenever the plate-holder is to be withdrawn from the apparatus. At present there seems no escape from this rather inconvenient procedure.

In order to secure radiations of the shortest wave-length, it is necessary to employ a disruptive discharge; under these electrical conditions the gas-filling is always more or less contaminated by material torn from the discharge tube. To reduce this cause of error, quartz has been employed in place of glass. A change of form as well as of material has also been made, for the internal capillary type, so well suited to some previous investigations,[1] cannot be employed when a violent disruptive discharge is used, because the propinquity of the end of the capillary to the inside of the spectroscope encourages a spreading of the discharge with a consequent fatal fogging of the photographic plate. The form of discharge tube finally arrived at was of the simple "end-on" type with a capillary 4.5 cm long and about 3 mm in diameter. The tungsten electrodes, provided by the kindness of the General Electric Company, were in the form of cylinders 1.5 cm long and about 3 mm in diameter. They were bound with fine tungsten wire to stouter wires of the same material which, in turn, were clamped in brass rods. These rods, covered with quartz sleeves, were contained in legs of the discharge tube and were sealed into these legs with Khotinski cement, the seals being kept hard by baths of mercury. The object of this arrangement was to restrict the electric discharge to the tungsten alone in the hope that thereby the contamination of the gas might be reduced to a minimum. The whole discharge tube thus constituted was sealed into a brass cone which, in turn, fitted air-tight into the cup on the face plate.

It is well known that tungsten if properly treated can be freed from occluded gases; this material, therefore, if employed for electrodes in a discharge tube, affords an escape from the contamination of the gas by the hydrogen which is so freely given off

[1] Lyman, *op. cit.*, p. 41.

by most metals. In practice, I have found tungsten far more satisfactory than any other material which I have employed, always provided that it has been freshly treated.

Atmospheric nitrogen, argon, hydrogen, and helium have been used in this investigation. I am indebted to my colleague, Professor Baxter, for the first-named gas. It was free from impurities save for those rare gases whose presence its atmospheric origin rendered inevitable.

I am indebted to the General Electric Company for my sample of argon. They informed me that the gas contained less than 1 per cent of impurity. As judged by spectroscopic observations in the visible spectrum, some hydrogen and a trace of nitrogen were present.

The hydrogen employed was prepared by electrolysis from amalgamated zinc and hydrochloric acid in an automatic generator of special type.[1] This method was recommended to me as one of the best available. After passing through towers of fused potassium hydroxide, the gas was collected over water. It was found necessary to prepare the hydrogen immediately before it was required for use, for, by standing over water, it is sure to become contaminated with air. Before being introduced into the phosphorous-pentoxide drying system, the gas was passed over platinized asbestos heated to redness in a quartz tube to insure the removal of the least trace of oxygen. A delicate test for the presence of a trace of air in the spectroscope is afforded by the appearance of the nitrogen lines near λ 5005 with disruptive excitation. Judged by the appearance of its visible spectrum the hydrogen seemed free from impurities.

The helium employed has been obtained from three sources. The first sample was prepared from clevite in the usual way and was purified over charcoal plunged in liquid air; the second was obtained from Professor Cady, of the University of Kansas; the third I owe to the generosity of Professor Boltwood. This last specimen I treated with pyrogallic acid and then passed over hot copper, then over hot calcium, and finally it was exposed to charcoal and liquid air. All the specimens were free from nitrogen

[1] Cook and Richards, *Proceedings of the American Academy*, 23, 149, 1887.

and oxides of carbon, but contained a trace of hydrogen. Charcoal cannot be trusted to remove the last remnant of this gas, but as the helium was bound to be exposed to some admixture of hydrogen from the walls of the spectroscope and from the discharge tube, it seemed useless to go into the greatest refinement of purification at the present stage of the investigation. It must be noted that after the gas had been admitted to the spectroscope it sometimes showed a trace of oxides of carbon, but it appeared to be free from nitrogen.

About 100 cc of helium are necessary for a single experiment; part of this gas is used in washing the spectroscope and part in the final observations.

In order to guard the spectroscope from mercury and to reduce the presence of hydrocarbons as far as possible, a U-tube with legs, each about 35 cm long, was plunged in liquid air and interposed between the spectroscope receiver and the rest of the system; this latter consisted of the gas reservoir, the McLeod gauge, and the Trimount oil pump.

When a disruptive discharge was required, the current was obtained from a ½ kw Clapp-Eastham transformer run from a 60-cycle, 110-volt circuit. A capacity of 0.014 microfarad was placed across the terminals of the secondary, while a spark-gap of about 3 mm was used in series with the discharge tube. Under these circumstances, the equivalent spark-gap between needle-points at the discharge tube was 8.5 mm. When the non-disruptive discharge was employed, the current through the tube was of the order of 15 milliamperes.

With the idea of eliminating altogether the difficulties which arise from the introduction of electrodes into a radiating gas, some attempts have been made to use the electrodeless discharge. In this experiment a Chaffee gap[1] was connected in series with the primary of a Tesla coil, the arrangement thus formed being shunted by a condenser which, in turn, was placed across the leads of a 500-volt circuit, one of the lines of which contained a choking coil. The capillary discharge tube was provided with armatures of tinfoil at either end and these were connected to the secondary of the

[1] E. L. Chaffee, *Proceedings of the American Academy*, **47**, 270, 1911.

Tesla. Thanks to the regular behavior of the Chaffee gap, a very brilliant and steady illumination resulted.

The successful reconnoissance of Saunders[1] into the region of extremely short wave-lengths suggested the advisability of trying a calcium arc in quartz. The arrangement was very similar to that described by Saunders except that the lamp was operated from the 500-volt circuit. The chief technical difficulty arises from the rapid deterioration of the quartz. In these experiments magnesium as well as calcium electrodes were employed.

If of the many experimental difficulties of the subject there is one which seems at present more troublesome than the others, and to which the limit of the spectrum may be directly traced, it is a luminosity in the gas content of the spectroscope excited by the disruptive discharge. The phenomenon makes its appearance when the pressure is of the order of 1 mm and produces a background of fog on the photographic plate in which faint lines are lost. This luminescence is especially aggravating since the low pressure at which it appears seems particularly favorable to the production of the lines of the very shortest wave-length. As a result of this difficulty long exposures are out of the question. It seems probable that if a window could be interposed between the discharge tube and the spectroscope, the excited luminosity would vanish.

Unfortunately, material for such a window is not at hand. A test with a bit of fluorite 1 mm thick, of proved transparency in the Schumann region, showed that this substance was opaque from λ 1230 to the extreme end of the spectrum at λ 600. A similar test with quartz 0.5 mm thick demonstrated that this substance shows no signs of transparency in the extreme ultra-violet.

Lenard[2] has interpreted some of his experiments on the ionizing power of light from an aluminum spark as proving the existence of radiations from this source lying on the more refrangible side of the quartz absorption band and transmitted to some extent by air. I have made an attempt to find these radiations by placing an aluminum spark in air outside my spectroscope and about 2 mm

[1] *Astrophysical Journal*, 40, 377, 1914.

[2] Lyman, *op. cit.*, p. 107.

distant from a quartz window 0.2 mm thick which closed the apparatus. The result of the experiment was entirely negative, the adjustment of the apparatus being such that the field covered by the photographic plate extended to λ 250.

After this account of the apparatus and the methods employed, it seems important to discuss the various spectra in some detail, paying particular attention to the circumstances under which they were produced.

The electrodeless discharge is easily dismissed; no radiation on the more refrangible side of λ 800 has been obtained with it. The arc discharge in quartz, both when calcium electrodes were used and when magnesium was employed, showed no lines more refrangible than λ 1000 which could be certainly attributed to these metals. The spectra in both cases consisted mainly of secondary hydrogen lines; the primary lines at λ 1216 and λ 1026 were, however, quite strong. The spectrum, which is intense, terminates near λ 905.

The absence of metallic lines in this region is also confirmed by experiments with the spark-discharge at reduced pressures. Pointed terminals of both aluminum and iron, about 1 cm apart, were arranged in a small globular vessel communicating directly with the spectroscope. A spark-gap in air, in series with the terminals, insured the disruptive nature of the discharge. At a pressure of about 1 cm of hydrogen the spectra obtained with both metals were characteristic of the gas-filling. They contained no lines which could be ascribed with certainty to either aluminum or iron. The spectra terminated near λ 1030. The amount of gas present in the light-path was about equivalent to that in a column of hydrogen at atmospheric pressure 2.5 cm long. The experiment, therefore, is of some interest as giving an idea of the order of transparency of hydrogen for these short wavelengths.

The relation between the spectra of helium and of hydrogen forms a fascinating subject for speculation, and the spectral region now under consideration is an excellent field for the test of hypotheses. However, as has already been pointed out, the data at hand cannot be made to yield conclusive answers to the questions

involved. The difficulty is inherent in the nature of the problem since the type of the apparatus which must be employed, if the region in question is to be studied at all, precludes the separation of the effects of absorption from those produced by radiation, and at the same time renders the elimination of traces of hydrogen from an atmosphere of helium extremely difficult. In considering the spectra obtained from the discharge tube, therefore, it must suffice for the present at least, to confine the attention to their general character.

On comparing the spectra obtained from hydrogen excited with and without capacity, it becomes evident that when the lines obtained in the latter case are subtracted from those produced with a disruptive discharge, some seventeen strong lines remain. Of these, λλ 1216, 1026, and 972 form the Balmer analogue predicted by Ritz; λ 1216 is one of the strongest lines in the spectrum; the second member, however, is so feeble as to be hardly visible in spectrum *a*, Plate III, but it is easily seen in spectrum *d*. This illustrates the curious fact, to which reference has been already made, namely, that the extreme lines of the Ritz series appear to be produced more strongly in helium with a non-disruptive discharge than in hydrogen when a condenser and spark-gap are employed. The line λ 972 is not visible in spectrum *a* but may be seen quite clearly on the original negative from which spectrum *d* was taken. In connection with Bohr's speculations it is important to observe that λ 1216, which forms the first member of the Ritz series, occupies exactly the same position when obtained from helium as when it is produced in hydrogen.

The striking pair of strong lines near λ 1086 and the wider pair near λ 992 have already been attributed to an impurity. They occur in both quartz and glass discharge tubes and their appearance is independent of the nature of the electrodes. They occur very strongly in nitrogen and may perhaps be attributed to this gas, though not with perfect certainty. They may possibly be produced by an oxide of carbon, a trace of which has been occasionally detected in the visible spectrum of helium and which probably takes its origin from the wax used to seal the spectroscope. An inspection of spectra *a* and *b* will show that these lines are less

intense in helium than in hydrogen; they cannot, therefore, be ascribed to helium, though Bohr has hinted that they belong to this gas.[1] The line at λ 1176 is perhaps the strongest in the whole hydrogen spectrum; it is equally strong in helium and is very strong in nitrogen; of its origin nothing positive can be said. All the remaining lines to λ 977 are stronger in nitrogen than in either helium or hydrogen; all occur in argon. From λ 997 to λ 904 all the lines, with one exception, occur in hydrogen, helium, and argon, but with relative intensities depending on the gas in which they are produced. All the lines on the more refrangible side of λ 900 are obtained only when helium is employed, with the exception of λ 833 which occurs in argon. The strength of the pair near λ 835 is striking. An examination of the print from which Plate III was made showed in spectrum *b*, if a magnifying glass was used, the extreme line near λ 600 A; however, this line is probably lost in the reproduction.

Nitrogen yields a few strong lines with a disruptive discharge besides those which it appears to contribute to the hydrogen spectra; with a non-disruptive discharge it yields but two or three weak lines near λ 1200. As its spectrum does not extend beyond λ 977, I have been chiefly interested in the gas in its character of an impurity and have made no measurements upon its lines.

As has been mentioned already, argon gives a spectrum containing many lines terminating only near λ 800, but here, again, I have not delayed the progress of this research in order to make measurements. A careful study of the argon spectrum in the future, however, will probably well repay the trouble.

The wave-lengths which are to be found in Table I were obtained by the two-slit method which I have frequently employed.[2] They rest on the hydrogen line λ 1216.0 and upon the shifted spectra of iron and aluminum. They make no claim to extreme accuracy, but I hope that, when standard wave-lengths shall have been established in this region, the values given in the table will not be found to depart from the standards by more than one unit.

[1] *Philosophical Magazine*, **30**, 401, 1915, note.

[2] Lyman, *op. cit.*, p. 45.

The numbers in the first column indicate the intensities of the lines as they occur in helium.

It must be remembered that λλ 1216, 1026, and 972 represent the only strong radiations on the less refrangible side of λ 900 which can be attributed to helium or to hydrogen with any degree of certainty. Even the extreme lines produced in helium alone may owe their appearance on the photographic plate to the superior transparency of the gas and may be produced by some subtle impurity.

TABLE I

STRONG LINES IN THE EXTREME ULTRA-VIOLET

Intensity in Helium	λ	Intensity in Helium	λ
1	599.0	3	992.0
1	643.7	4	1010.6
2	702.4	4	1026.0
3	703.5	5	1037.0
5	718.2	2	1084.9
2	796.8	5	1086.1
8	833.4	1	1134.7
7	834.8	8	1175.5
4	904.6	10	1176.3
2	916.7	5	1199.8
1	972.7	10	1216.0
2	976.8	1	1236.0
6	977.6	5	1247.9
1	990.2		

The result of this investigation is easily stated: the spectrum has been extended to λ 600 A.

I cannot conclude this article without expressing my appreciation of the skill and patience which my assistant, Dr. Paul Sabine, has shown during the whole course of this research.

THE JEFFERSON LABORATORY
CAMBRIDGE
December 1915

THE STELLAR MAGNITUDES OF THE SUN, MOON, AND PLANETS

By HENRY NORRIS RUSSELL

An examination of the data concerning the brightness and albedo of the various members of the solar system, begun with the object of obtaining trustworthy values for textbook purposes, has shown the desirability of a general revision of the data, and discussion of the theory. The resulting conclusions regarding the brightness of the sun, moon, and planets are presented below. The data bearing on the albedo of the earth, and the theoretical discussion of the whole will form a later paper.

I. THE STELLAR MAGNITUDE OF THE SUN

Several determinations of this fundamental constant have been made in the last few years, and its value is now known with considerable accuracy.

a) Of the older determinations,[1] the only one which now deserves serious consideration is that by Zöllner,[2] who used a photometer of the type still called by his name, and compared the starlike image of the sun formed by a suitable combination of lenses and shade-glasses, and the direct image of Capella formed by a larger objective, with the same "artificial star." The corrections for the loss of light in the optical systems and shade-glasses were very carefully determined. The sun was observed on 6 days, and the star on 11 nights, with remarkably accordant results, the average deviation of a single day's or night's result from the general mean being only $0^{M}05$, which is all the more notable because the light of the artificial star was assumed to be absolutely constant throughout the interval of four months covered by the observations, and any variations in it would tend to increase the deviations. He concludes that the difference of magnitude between the sun and Capella is

[1] See Müller, *Die Photometrie der Gestirne* (Leipzig, 1897), p. 316.

[2] *Photometrische Untersuchungen* (Leipzig, 1865), pp. 107, 124–25.

26.87 ± 0.05, the uncertainties of the instrumental constants being included in the assigned probable error. With the Harvard magnitude of Capella, 0.21, the magnitude of the sun would be -26.66.

b) Next in order of time is the determination by C. Fabry,[1] who compared the light of a standard electric lamp, shining through a blue screen, with that of the sun weakened by divergence through a lens of short focal length by means of a Lummer-Brodhun photometer, and later compared the light of the same standard, reduced to a stellar image by means of a microscope objective and reflected from a glass plate at an angle of 45°, with Vega, directly seen by the naked eye. Under clear skies and for bodies near the zenith the results obtained on different days or nights differed only by *quelques centièmes*. Fabry also determined the candle-power of his lamp and the percentage of its light transmitted by the blue solution, and gives as his results that the sun in the zenith and at mean distance gives light equal to 100,000 *bougies décimales* at 1 m, while Vega in the zenith is equal to a candle at 780 m. This makes the sun $26^{M}.94$ brighter than Vega, or of magnitude -26.80 on the Harvard scale.

c) Ceraski,[2] observing at Moscow in 1903, compared with a Zöllner photometer the image of the sun, reflected from the surface of a small lens at a distance of 152 m, with Venus, which was near elongation and visible in full daylight. Later auxiliary comparisons were made between Venus and various bright stars. The constants of reduction were very carefully determined. Observations on seven days made the sun $22^{M}.93$ brighter than Venus 112 days after inferior conjunction, with an average deviation of $0^{M}.10$. The supplementary observations, made more than a year later, showed that Venus, 42 days before inferior conjunction (when, according to Müller, it was $0^{M}.66$ brighter than on the former occasion), was $4^{M}.86$ brighter than Procyon, $6^{M}.30$ brighter than Polaris, and $3^{M}.22$ brighter than Sirius. With the Harvard magnitudes of these stars, and weights 4, 4, 1, proportional to the number of

[1] *Comptes Rendus*, **137**, 973 and 1242, 1903. The more detailed account (*Assoc. française pour l'Avancement de Science, Congrès d'Angers*, **1903**, p. 255) is inaccessible to the writer.

[2] *Annales de l'Observatoire de Moscou*, 2d series, **5**, 1–30, 1911.

nights of observation, the sun's magnitude comes out −26.60 on the Harvard scale.

d) W. H. Pickering,[1] observing in Jamaica in 1901, with a 12-inch telescope of 135 feet focal length, compared, by means of a shadow photometer, the brightness of the center of the focal image of the sun, shining through a small aperture, about 1 mm in diameter, in front of the objective, and of the extra-focal images of stars, 2 or 3 mm in diameter. The comparison light was a pentane lamp shining through a cell filled with a blue solution. Ten independent determinations, on as many days and nights, using Capella, Arcturus, Vega, and Sirius as reference stars, gave the sun's magnitude as −26.83, with an average deviation of $0^{M}.19$ for a single determination.

These four determinations, made in altogether different ways, are in remarkably good agreement, especially when it is considered what enormous instrumental constants, representing the effects of the various methods of weakening the sun's light, have to be determined in every case. The results on the Harvard scale are:

Zöllner	$-26^{M}.66$
Fabry	−26.80
Pickering	−26.83
Ceraski	−26.60
Simple mean	-26.72 ± 0.04

The probable error of one determination, assuming them to be of equal weight, comes out $\pm 0^{M}.075$, showing that the systematic errors of the various methods of observation must be very small, for it is highly improbable that four such radically different processes should all be affected with errors of the same sign and magnitude.

Two determinations of the sun's photographic magnitude have also been made, each based on a considerable amount of careful work.

King,[2] comparing the sun with Polaris, Arcturus, and Capella by his well-known extra-focal method, finds a mean value of $-25^{M}.83$,

[1] *Harvard Annals*, **61**, 56–71.

[2] *Ibid.*, **59**, 248.

with an average deviation of only $0^{M}.07$ for his 11 determinations. The probable error of the mean is presumably somewhat larger than this agreement would indicate, on account of the uncertainty of the measurements of the absorption of the screens which were employed.

Birck,[1] working extra-focally by a method essentially similar to that of Pickering's visual observations, finds differences in photographic magnitude between the sun and stars which, with King's photographic magnitudes for the latter, give the following showing:

Star	Difference of Magnitude	Color-Index		Photographic Magnitude	Photographic Magnitude of Sun	
	Birck	Birck	King	King	Uncorrected	Corrected
Vega	25.95	0.00	0.00	0.14	−25.81	−26.15
Capella	27.04	1.02	0.82	1.03	−26.01	−26.01
Arcturus	27.76	1.71	1.09	1.33	−26.43	−26.20

The discordance between the results found from the three stars arises from a difference in color-equation between King's and Birck's methods of observation. The photographic difference in magnitude between Vega and the other stars can be deduced directly from Birck's observations, and comparison with the Harvard visual magnitudes gives the color-indices in the third column, which indicate a variation with spectral class about 50 per cent greater than for King. If Birck's results are altered by one-third of the differences of his color-indices from that of Capella, and so approximately reduced to the Harvard photographic scale and the solar type of spectrum, the figures given in the last column are obtained. These are in good agreement and give a mean of $-26^{M}.12$. On account of the uncertainty of this correction, King's determination may be given double weight in taking the mean, and the value $-25^{M}.93$ may be adopted for the sun's photographic magnitude on the Harvard scale.

The color-index of the sun, resulting from the visual and photographic magnitudes here adopted, is $+0^{M}.79$, which differs from

[1] Inaugural dissertation (Göttingen, 1909).

King's mean value[1] for stars of Class G, $+0^{M}.72$, by less than its probable error.

II. THE MAGNITUDES OF THE PLANETS

By far the most extensive and homogeneous series of observations of the brightness of the planets are those of Müller,[2] which will be adopted as the main basis of the present discussion. The scale of magnitudes to which these observations are referred is defined by Table IV, p. 235, of his memoir, and is intermediate between those of the *Revised Harvard Photometry* and the *Potsdam Durchmusterung*. Müller's magnitudes for his 14 fundamental stars, which range in brightness from Sirius to Polaris, are all brighter by $0^{M}.19$ than those of the final Potsdam catalogue, and the differences for the fainter stars, though more irregular, average about the same. The mean values of the difference Müller minus Harvard, grouped according to spectral class and magnitude, are given in Table I. The relative color-equation of the two systems is

TABLE I

	Spectral Class						
	B	A	F	G	K	K2 to M	All
Brighter than $4^{M}.0$							
Number	3	6	3	3	3	2	20
Mean M−H	$+0^{M}.08$	$+0^{M}.18$	$+0^{M}.07$	$-0^{M}.02$	$+0^{M}.02$	$-0^{M}.13$	$+0^{M}.06$
Fainter than $4^{M}.0$							
Number	3	5	3	2	8	5	26
Mean M−H	$+0^{M}.04$	$+0^{M}.01$	$-0^{M}.08$	$-0^{M}.15$	$-0^{M}.10$	$-0^{M}.06$	$-0^{M}.06$
Magnitude	$0^{M}.3$	$1^{M}.9$	$3^{M}.7$	$5^{M}.0$	$5^{M}.5$	$6^{M}.2$	$7^{M}.2$
Number	8	8	8	9	8	8	9
Mean M−H	$+0^{M}.05$	$+0^{M}.07$	$-0^{M}.05$	$-0^{M}.02$	$-0^{M}.08$	$-0^{M}.10$	$-0^{M}.01$

small, being approximately $0^{M}.035$ for each spectral class, Müller making the red stars the brighter. If the observed differences are reduced to spectrum G on this assumption, and grouped according to magnitude, the means given in the lower part of Table I are obtained. There may be a difference in scale between the two

[1] *Harvard Annals*, **59**, 179.

[2] *Potsdam Publications*, **8**, Part IV, pp. 197–371, 1893, with some additions in *Die Photometrie der Gestirne*, 1897.

systems, but it is very small. In what follows it will be assumed that the brighter planets are measured $0^{M}.06$ fainter by Müller than on the Harvard system, and that for Uranus, Neptune, and the asteroids no correction is necessary.

For those planets which show conspicuous phases, Müller's formulae for the stellar magnitude corresponding to the phase-angle, and reduced to mean distance from the sun and unit distance from the earth, in the case of Mercury and Venus, and to mean opposition for Mars, are:

Mercury.... I. Mag. $= -1.04 + 0.0368(\alpha - 50°)$
II. Mag. $= -0.90 + 0.0284(\alpha - 50°) + 0.000102(\alpha - 50°)^2$
Venus..... Mag. $= -4.71 + 0.01322\alpha + 0.000000425\alpha^3$
Mars..... Mag. $= -1.79 + 0.0149\alpha$

Mercury can be observed only between phase-angles 50° and 120°, and within this interval the two formulae represent the observations almost equally well, the probable error of the unit of weight being $\pm 0^{M}.137$ with I, and $\pm 0^{M}.135$ with II. An observation by Jost[1] during the total solar eclipse of May 28, 1900, at phase-angle 6°.8, gave the value -2.81 for the magnitude of the planet, reduced to standard distance. Formulae I and II give -2.63 and -1.94, respectively. Since, however, formula I leads to the rather improbable conclusion that Mercury is 21 times brighter at the full than at the half phase, both formulae will be retained. As Müller points out, formula II indicates a phase-variation almost exactly like that of the moon.

The formula for Venus represents the observations very satisfactorily over the wide range from 24° to 156°, and that for Mars is equally good from 0° to 47°, the maximum phase observable. Within the range of phase in which both these planets have been observed, their variation is almost identical. Jupiter can be observed only up to about $\alpha = 12°$. Within this region its brightness varies far less than that of any of the inner planets would. Müller's mean results for groups of about 30 observations are as follows:

Phase-angle	1°.0	3°.1	5°.5	7°.5	9°.0	10°.3	11°.1
Magnitude	−2.27	−2.23	−2.23	−2.24	−2.20	−2.26	−2.20

[1] *Heidelberg Mittheilungen*, No. 1 (1901).

The whole variation within these limits can hardly much exceed $0^{M}.05$, whereas if the planet behaved like Mars it would be $0^{M}.15$, and if like the moon, $0^{M}.22$. Lambert's law gives a variation of $0^{M}.02$ and the Lommel-Seeliger law $0^{M}.03$.

Saturn, when the ring is invisible, likewise shows no sensible variation with phase, according to Baldwin's observations.[1] The phase-variations which appear when the ring is visible are explicable by Seeliger's theory, which takes account of the meteoric constitution of the rings. Müller, applying this theory to his 252 observations,[2] finds for the magnitude at mean opposition with ring invisible the value +0.95, in exact agreement with the results of Baldwin's direct observations. The correction for the light of the rings is approximately $0^{M}.04$ for every degree of elevation of the earth above their plane.[3]

For the magnitude of Uranus at mean opposition, Müller finds the value 5.86 from 93 observations in 8 different years, while Pickering,[4] from 31 nights' measures in 5 years, obtains 5.51, a surprising discordance. The value 5.74 will be adopted here, giving Müller's observations double weight on account of their greater number. For Neptune, Müller finds 7.66 as the mean of 138 observations, Pickering[5] 7.65 from 66 observations, and Baldwin[6] 7.99 on the Potsdam scale, or 7.64 on the Harvard scale, from 32 observations—all in remarkable agreement.

For the four brightest asteroids, whose diameters have been determined by Barnard, the magnitudes determined by Pickering[7] from the results of several observers have been adopted. The rates of change in magnitude per degree of phase-angle are from Müller's summary,[8] taking again the mean result of all observers. They are 0.043 for Ceres, 0.038 for Pallas, 0.030 for Juno, and 0.022 for Vesta. The mean value of this constant for 34 asteroids is found by Müller to be $0^{M}.030$ per degree.

[1] *Monthly Notices*, **69**, 458, 1909.
[2] *Op. cit.*, p. 348.
[3] *Photometrie der Gestirne*, p. 347.
[4] *Harvard Annals*, **46**, 204.
[5] *Ibid.*, **24**, 265; **46**, 203.
[6] *Monthly Notices*, **68**, 620, 1908.
[7] *Harvard Circular*, No. 169, 1911.
[8] *Photometrie der Gestirne*, p. 378.

Guthnick's magnitudes[1] for the four Galilean satellites of Jupiter have been adopted, and for Titan the mean of the results of Guthnick and Wendell,[2] which differ by only $0^{M}.03$. Nothing at all is known about the change of brightness with phase for any of these satellites.

The adopted values of the stellar magnitude at mean opposition, on the Harvard scale, for the bodies more remote than Mars, are therefore:

Jupiter .....−2.29	Ceres.....+7.15	Jupiter I.....+5.54
Saturn.....+0.89	Pallas.....+7.84	Jupiter II.....+5.69
Uranus .....+5.74	Juno.....+8.95	Jupiter III....+5.08
Neptune.....+7.65	Vesta.....+6.04	Jupiter IV ....+6.26
		Titan.....+8.30

For the photographic magnitudes of the planets, King's extrafocal observations[3] appear to be the only published material. King has derived empirical formulae from his own observations to represent the variation with phase in the case of Venus, Mars, and Saturn, and derived color-indices by comparing these magnitudes, extrapolated to zero phase, with Müller's visual magnitudes, similarly extrapolated, applying to the latter a correction of $-0^{M}.12$ to reduce from the Potsdam to the Harvard scale. It appears to the writer that it would be preferable to compare directly the visual and photographic observations at the phases at which they were actually made. As Müller's observations are much the more numerous, and his empirical formulae represent their course very closely, these were adopted as a standard of reduction, and the differences between King's photographic magnitudes and these formulae were grouped according to the phase-angle, giving each night's observations equal weight, with the results shown in Table II. The magnitudes of Saturn computed by Müller according to Seeliger's theory were employed. There is no appearance of change in the color-index with phase, except in the case of the first group of observations on Venus, which are very discordant

[1] *Astronomische Nachrichten*, **198**, 251, 1914.

[2] *Harvard Annals*, **69**, 223, 1914.

[3] *Ibid.*, **59**, 261–64.

inter se. Further observations here are evidently desirable; in their absence the simple mean of all the observations is taken.

TABLE II

Venus				Mars			
Phase-Angle	Mean K−M	Obs.	Average Deviation	Phase-Angle	Mean K−M	Obs.	Average Deviation
64°.6	+1^{M}.10	8	0^{M}.35	4°.9	+1^{M}.30	8	0^{M}.21
75.4	+0.67	7	0.33	15.6	+1.34	10	0.18
84.9	+0.46	5	0.29	25.0	+1.33	10	0.12
116.6	+0.52	7	0.14	32.7	+1.31	8	0.26
Jupiter				Saturn			
2.6	+0.49	9	0.12	0.6	+1.07	6	0.06
8.6	+0.38	10	0.12	2.7	+1.05	6	0.06

After correction by 0^{M}.06 to reduce to the Harvard visual scale (as found above), the color-indices of the four planets become: Venus, +0.78; Mars, +1.38; Jupiter, +0.50; Saturn, +1.12.

III. THE VARIATIONS OF THE MOON'S BRIGHTNESS WITH PHASE

Seven series of observations of the relative brightness of the moon at different phases demand consideration. Though the light-curves derived by the various observers differ very considerably, their actual observations can all be satisfactorily represented by the same mean curve, as is shown below.

The earliest reliable observations are those of Sir John Herschel,[1] which consist of direct comparisons of many stars with the reduced image of the moon given by his "astrometer." Since they were inconsistent with Euler's "law," which Herschel supposed to be true, he concluded that they were affected by systematic errors, and their true value was first pointed out by Bond and Zöllner. Zöllner's reduction[2] of these observations was used in deriving the provisional curve described below, but in the final discussion the

[1] *Results of Astronomical Observations Made at the Cape of Good Hope* (London, 1847), pp. 353–374.

[2] From the plot in Tafel IV of his *Photometrische Untersuchungen*, with the corrections given in the footnote to p. 175.

values are taken from the new reduction of Herschel's observations which is described in the next section of this paper.

Bond,[1] who first showed that the actual phase-variation of the moon was very different from that predicted by the existing theories, compared the image of the moon reflected from a silvered glass globe with that of a lamp, shining through a small diaphragm and reflected in the same way, and varied in distance from the globe to obtain equality of the images. Of his 13 nights of observation (in the spring of 1860), 3 are described as "hazy," and the results then obtained have received half-weight in the present discussion, as have also a set of observations made when the moon's altitude was only 8°.

Zöllner's observations[2] were made with two polarizing photometers, and with exemplary care. He was the first to introduce a blue screen to make the artificial light similar in color to the moon. As in the case of his observations of the sun, the accuracy of his results depends on the constancy of his standard lamp for weeks at a time, but the agreement of his results shows that in this respect his apparatus was surprisingly successful.

The visual observations of W. H. Pickering[3] were made by comparing direct moonlight by means of a shadow photometer with a pentane lamp shining through a blue solution. Of the 26 nights' observations, those on 2 nights have been rejected by Professor Pickering, one on account of known instrumental defects, the other for a very large but unexplained discordance.

King's extensive series of observations[4] consist of photographic comparisons of moonlight with the extra-focal images of stars, without the intervention of artificial light. Their large number, 83, on 64 nights, adds to the value of the series.

Stebbins and Brown,[5] using selenium cells, compared the intensity of direct moonlight with that of an amyl-acetate standard lamp, the latter being put at such a distance as to secure equality

[1] *Memoirs of the American Academy of Arts and Sciences*, N.S. **8**, 221–266, 1861.

[2] *Photometrische Untersuchungen*, p. 102.

[3] *Harvard Annals*, **61**, 62–69.

[4] *Ibid.*, **59**, 63–94.

[5] *Astrophysical Journal*, **26**, 326, 1907.

of effect, thus eliminating the question of the proportionality of the electrical effect to the intensity of illumination. These observers were the first to call attention to the difference in brightness of the moon at equal phases before and after the full. The mean of all the results obtained on each night has been taken, and all nights given the same weight.

Finally, just in time for inclusion in the present discussion, has appeared Wirtz's reduction of the observations of Wislicenus,[1] the most extensive series so far published. They consist of comparisons with a Zöllner photometer of a reduced point image of the moon with Polaris. The normals given by Wirtz have been taken as they stand.

In reducing these observations to a common standard, each observer's results (reduced when necessary to differences of stellar magnitude from his assumed brightness of the full moon) were compared with a provisional light-curve (the mean of the curves given by Pickering, King, and Stebbins). It appeared at once that much the greater part of the discrepancies between the light-curves given by the various observers arose, not from contradictions in the observations, but from the way in which their curves had been drawn, especially in the immediate neighborhood of the full, and that the different series of observations could be brought into better agreement by applying corrections to the values of the full brightness assumed by each observer. As these values were determined independently in each case, from freehand curves or empirical formulae, and as the relation between the brightness of the moon and the ordinary standards of stellar magnitude, when determined at all, is known with much less accuracy than the relative brightness of the different phases, the application of these zero-point corrections to improve the agreement of the various series is entirely permissible.

After several approximations, during the course of which Wislicenus' important series of observations appeared and was included in the discussion, the light-curve given in Table III was adopted as definitive. In this table the first column gives the phase-angle; the second, the difference of the moon's stellar

[1] *Astronomische Nachrichten*, **201**, 289–331, 1915.

magnitude at this phase, before the full, and that of the full moon, the third, the corresponding intensity of moonlight, taking the full moon as 1000; and the fourth, the mean surface brightness of the visible portion of the moon's disk, expressed in stellar magnitudes and referred to the full as standard. The next three columns give similar data for the moon after the full; the following column, the mean magnitude for equal phases on both sides of the full; and that succeeding it, magnitudes computed on the empirical assumption that the brightness of the moon varies as the cube of its elongation from the sun. The residuals in the last column show that this relation is very nearly true except within 30° of the full, where it gives too small a value.

TABLE III

LIGHT-CURVE OF THE MOON

PHASE-ANGLE	BEFORE FULL			AFTER FULL			MEAN		
	Mag.	Light	Surface Bright-ness	Mag.	Light	Surface Bright-ness	Obs.	Comp.	O.−C.
0°	$0^{M}.00$	1000	$0^{M}.00$	$0^{M}.00$	1000	$0^{M}.00$	$0^{M}.00$	$0^{M}.16$	$-0^{M}.16$
10	0.22	816	0.21	0.22	816	0.21	0.22	0.34	− .12
20	0.44	666	0.41	0.48	642	0.45	0.46	0.54	− .08
30	0.67	540	0.59	0.74	505	0.66	0.72	0.75	− .03
40	0.90	436	0.77	1.03	387	0.90	0.96	0.97	− .01
50	1.13	353	0.92	1.31	299	1.10	1.22	1.23	− .01
60	1.37	283	1.06	1.58	234	1.27	1.47	1.47	.00
70	1.65	218	1.22	1.86	180	1.43	1.76	1.76	.00
80	1.98	161	1.40	2.17	136	1.59	2.07	2.07	.00
90	2.35	115	1.60	2.50	100	1.75	2.42	2.41	+ .01
100	2.78	77	1.82	2.86	72	1.90	2.82	2.79	+ .03
110	3.22	51	2.01	3.27	49	2.06	3.25	3.23	+ .02
120	3.77	31	2.26	3.74	32	2.23	3.75	3.74	+ .01
130	4.39	18	2.52	4.30	19	2.43	4.35	4.35	.00
140	5.14	9	2.81	4.98	10	2.65	5.06	5.06	.00
150	6.09	4	3.15	5.89	4	2.95	5.99	5.99	0.00

The manner in which this curve represents the observations of the various series is shown in Table IV, which gives, for means formed from small groups of the observations, the mean phase, the observer's initial, the number or weight of observations combined into the mean, and the residual from the curve. The agreement of the results of the different observers is further illustrated in Table V, in which are given the mean residuals for the observa-

tions near first quarter, full moon, and last quarter, and also the zero correction, described above, which has been applied to the magnitudes as given by each observer, the average number of observations combined into a mean in each case, and the average

TABLE IV

OBSERVED MEANS

Phase	Obs.	No.	O.−C.	Phase	Obs.	No.	O.−C.	Phase	Obs.	No.	O.−C.
−152°..	B	½	+1ᴹ.04	− 34°..	B	2	−0ᴹ.04	+ 34°..	Z	4	−0ᴹ.05
−146...	W	3	+0.06	32...	S	3	− .10	40...	H	3	.00
−139...	K	3	− .11	32...	H	3	+ .11	41...	W	4	+ .11
−138...	P	2	− .16	31...	K	5	+ .07				
								+ 55...	K	6	− .04
−134...	W	5	− .13	− 23...	Z	3	− .08	57...	W	5	+ .02
124...	B	3	− .07	18...	W	5	− .03	60...	S	3	+ .13
126...	K	5	− .19	14...	K	6	+ .15				
118...	W	4	+ .17					+ 61...	P	4	+ .01
				− 12...	K	8	+ .02	61...	Z	3	+ .12
−103...	W	4	− .12	11...	H	3	− .04	66...	W	3	− .19
101...	K	7	− .01	9...	W	3	− .06				
98...	S	2	+ .13					+ 78...	W	4	+ .05
98...	W	3	+ .13	− 8...	S	1	− .10	80...	K	4	− .16
				7...	Z	3	− .06	82...	H	1	− .15
− 95...	P	1	+ .15	5...	B	1½	− .01	84...	S	2	− .14
83...	B	2½	+ .05	4...	K	6	− .04	93...	W	3	+ .02
82...	W	5	+ .06	− 2...	P	1	+ .22				
79...	K	7	.00					+ 95...	P	3	+ .22
71...	W	4	.00	+ 3...	W	6	− .02	100...	W	5	+ .04
				6...	P	3	− .07	102...	S	2	− .12
− 67...	S	3	− .11	7...	S	1	− .03	103...	K	4	+ .15
65...	K	6	− .08								
64...	H	3	− .13	+ 8...	K	4	+ .33	+111...	W	5	− .07
				8...	Z	2	− .01	124...	W	3	− .21
− 58...	Z	3	+ .09	13...	B	1½	− .18	125...	P	1	+ .18
58...	W	6	+ .03	14...	P	2	− .02	127...	S	1	+ .42
50...	K	6	− .01	16...	W	5	− .01				
								+133...	K	1	− .66
− 49...	H	4	+ .04	+ 21...	P	3	− .13	139...	W	6	+0.04
40...	W	6	− .04	22...	H	2	− .03				
35...	Z	4	0.00	26...	K	5	+ .05				
				+ 28...	S	3	+ .06				
				28...	W	5	+ .16				
				30...	P	4	−0.05				

of the residuals given in Table IV, after reduction to this mean weight. There is very little evidence of any systematic difference between the results of the different observers, though King's photographic observations may perhaps indicate a smaller range between the full and half moon than the others do. It seems legitimate,

therefore, to combine the results of the different observers into more comprehensive normals. Unit weight was given to each individual observation by Bond, King, or Wislicenus, and weight 2 to each observation by the others, these weights being based on the average residuals given in Table V, and also on the general reliability of the

TABLE V

Observer	Zero Correction	Mean Residuals (Algebraic)						Average Deviation	
		First Quarter		Full		Last Quarter			
		Mag.	Obs.	Mag.	Obs.	Mag.	Obs.	Mag.	Obs.
Herschel	+0.00	−0.04	7	+0.02	8	−0.04	4	±0.07	2.7
Bond	+ .24	+ .08	6	− .07	5	.....	.....	± .14	1.9
Zöllner	+ .01	+ .04	7	− .06	8	+ .02	7	± .06	3.1
Pickering	+ .01	− .06	3	− .05	13	+ .11	8	± .11	2.4
King	+ .25	− .06	34	+ .08	40	− .06	15	± .09	6.5
Stebbins	− .02	− .01	5	− .03	8	+ .04	8	± .12	2.1
Wislicenus	0.00	+0.02	34	+0.01	34	−0.02	34	±0.07	4.4

methods of observation. The grouping of the observed means is shown by the spacing of Table III, and the resulting normals are given in Table VI. The residuals from the adopted light-curve are apparently accidental in character. Their average value, regardless of sign, is only ±0.035.

TABLE VI

Normal Magnitudes

Phase	Magnitude	O.−C.	Weight	Phase	Magnitude	O.−C.	Weight
−142°	5.32	+0.01	8½	+ 5°	0.07	−0.04	14
−126	4.07	− .06	17	+ 12	0.28	+ .01	18½
−100	2.80	+ .02	18	+ 23	0.52	− .04	15
− 80	2.02	+ .04	20½	+ 29	0.75	+ .04	19
− 65	1.40	− .11	18	+ 38	0.97	.00	18
− 55	1.29	+ .04	18	+ 57	1.54	+ .04	17
− 42	0.95	.00	22	+ 62	1.65	+ .01	17
− 32	0.74	+ .02	19	+ 83	2.20	− .07	17
− 18	0.41	+ .01	17	+100	2.95	+ .09	19
− 11	0.22	− .02	17	+119	3.71	+ .02	12
− 5	0.09	−0.02	17½	+138	4.76	−0.06	7

Tables III and IV are illustrated graphically in Fig. 1. It is clear that the transition from increase to decrease of brightness in

passing through the full is almost abrupt. The visual stellar magnitude of the moon at any phase may be read directly from this figure.

The variation of the moon's brightness with phase is now among the best determined of photometric data, and the values given in Table III are probably within $0^{M}.05$ of the truth up to phase-angles of about 135°. Extension of the curve toward the new moon is necessarily uncertain, but the assumption that the moon's light continues to vary nearly as the cube of the elongation from the

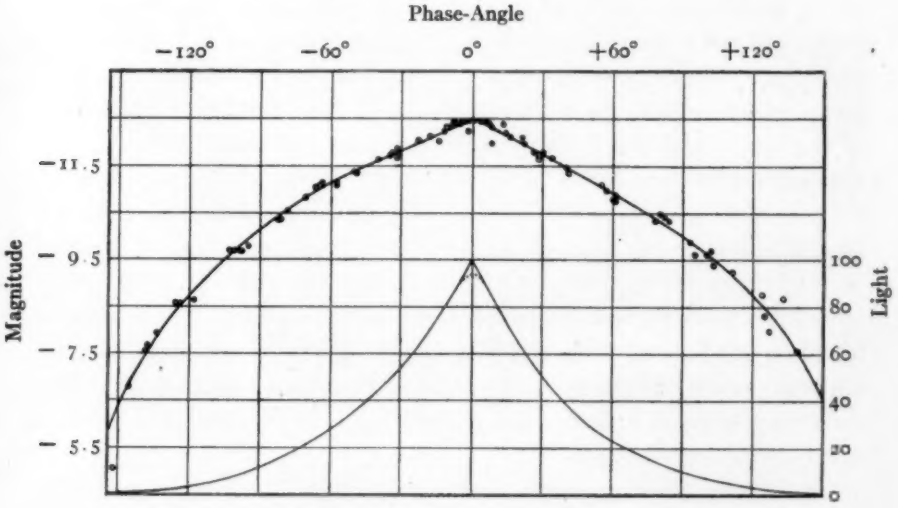


FIG. 1.—Light-curve of the moon. The upper curve shows the variations in stellar magnitude; the lower, those of the actual intensity of moonlight.

sun is probably as good a guide as any. According to this table, the full moon is 8.7 times as bright as the half moon at the first quarter, and 10.0 times as bright as the last quarter. The difference of the brightness at the opposite quadratures, as Stebbins points out, is immediately explained by the greater extent of the dark *maria* in the eastern half of the visible disk. The great difference between the light of the full and half phases arises, as is well known, from the rough character of the lunar surface, which, except at the full, is largely darkened by the shadows of its own irregularities.

IV. THE STELLAR MAGNITUDE OF THE FULL MOON

While the results of different observers agree closely with regard to the relative brightness of the moon's phases, their determinations of the absolute brightness of the mean full moon are extraordinarily discordant, and this important constant is still decidedly uncertain. The various series of observations will here be discussed in historical order.

a) *Herschel's observations—new reduction.*—These observations,[1] made at the Cape of Good Hope in 1836, consist of 165 comparisons, on 19 nights, of a reduced image of the moon with the brighter stars, and were made primarily with the intention of determining the relative brightness of these stars. They form, however, the most extensive and direct comparisons of the relative brightness of the moon and stars which had ever been published before the appearance of Wislicenus' work in 1915. Herschel derived from his comparisons with α Centauri a value of the ratio of the brightness of the moon to that of this star, and Bond and Zöllner have derived from his observations data concerning the variations of moonlight with phase; but no general reduction of the measures, based on modern photometric magnitudes of the comparison stars, appears ever to have been made. Such a reduction (which involved no serious labor) has been carried out by the writer, and its results are briefly presented here.

Herschel's "astrometer," with which these observations were made, consisted essentially of a long staff which could be pointed at any star, on which was mounted a movable slider, carrying a lens of short focal length, into which the moon's light was reflected through a right-angled prism. By varying the distance of the slider from the eye, the starlike image of the moon produced by the lens was made equal in brightness to the star, viewed without optical aid. This simple method of observation yielded remarkably accurate results, as Zöllner has shown,[2] 9 nights' measures of α and β Centauri giving their difference of magnitude with an average residual of $\pm 0^{M}.06$ from the mean.

[1] *Results of Astronomical Observations Made at the Cape of Good Hope* (1847), pp. 353–372.

[2] *Photometrische Untersuchungen*, p. 176.

From each measured distance d (in inches) at which the moon's image appeared equal to a star of magnitude m' the apparent magnitude m of this image as seen at the standard distance of 100 inches was derived by the equation

$$m = m' + 10 + 5 \log d$$

and corrections were applied for the atmospheric extinction (which were usually small, as the observations were made at high altitudes). These corrections were taken from a nomogram prepared in the manner described by Birck,[1] and based on the Potsdam extinction table, which made their determination very rapid.

The details of the reductions for a typical night are given in Table VII. The magnitudes of the stars are taken from the

TABLE VII

REDUCTION OF HERSCHEL'S SERIES K

STAR	d	OBSERVATION	m'	EXTINCTION		m	RESIDUAL
				Moon	Star		
α Centauri	33.3	6	0.06	0.01	0.02	2.46	+0.17
α Bootis	38.7	4	0.24	.01	.16	2.45	+ .16
α Lyrae	51.7	3	0.12	.05	.59	2.00	(−0.29)
β Centauri	55.0	5	0.86	.01	.02	2.17	−. .12
α Aquilae	61.5	2	0.89	.05	.19	2.09	− .20
α Triang. Aust	80.7	4	1.88	.05	.05	2.35	+ .06
α Pavonis	94.0	3	2.12	.05	.07	2.28	− .01
γ Centauri	107.3	3	2.38	.01	.03	2.24	− .05
β Corvi	131.0	2	2.84	0.01	0.04	2.28	− .01
Mean (excluding α Lyrae)	.....	.....	.....	0.02	0.07	2.29	±0.10

Revised Harvard Photometry. The observations of α Lyrae, which was at an altitude of only 16°, are excluded from the mean. The average deviation of the observations from their mean is rather smaller than usual, the mean value of this quantity for the 14 nights on which five or more stars were observed being $\pm 0^{M}.14$. The moon's declination was $-17°$, which explains why the extinction was usually smaller for it than for the stars.

The results thus obtained for each night must be reduced to mean distance, and then to full moon. Herschel gives, for each

[1] Inaugural dissertation (Göttingen, 1909), p. 54.

night, the theoretical brightness of the moon according to Euler's law, computed by the formula

$$M = \frac{1000\Delta^2}{(R \times 933''.5)^2} \sin^2 \tfrac{1}{2} e \qquad (1)$$

where Δ is the moon's apparent semi-diameter, including the augmentation, R the earth's radius vector, $933''.5$ the moon's mean semi-diameter, and e its elongation from the sun. If the last factor be omitted, this gives the correction to mean distance.

TABLE VIII

Date	Stars	Average Deviation	m	Phase-Angle	Correction to Mean Distance	Correction to Full Moon	Image of Full Moon	Wt.	Residual
1836									
March 28.....	17	±0.M17	2.M12	−50°	−0.M05	−1.M14	0.M93	4	+0.M22
30.....	17	.16	1.56	−25	+ .01	−0.57	1.00	4	+ .29
31.....	4	.12	0.93	−13	+ .04	−0.29	0.68	2	− .03
April 1.....	5	.08	0.45	− 1	+ .06	−0.01	0.50	2	− .21
3.....	13	.11	1.19	+28	+ .10	−0.69	0.60	3	− .11
4.....	12	.15	1.61	+41	+ .09	−1.07	0.63	3	− .08
7.....	5	.16	2.71	+82	+ .08	−2.23	0.56	2	− .15
26.....	13	.13	2.06	−59	− .03	−1.35	0.68	3	− .03
27.....	19	.12	1.73	−46	+ .01	−1.03	0.71	4	.00
June 29.....	5	.17	1.00	+13	+ .11	−0.30	0.81	2	+ .10
July 22.....	8	.10	2.29	−75	+ .03	−1.81	0.51	2	− .20
24.....	8	.14	1.58	−47	+ .09	−1.06	0.61	2	− .10
26.....	13	.16	1.10	−19	+ .10	−0.42	0.78	3	+ .07
Aug. 22.....	3	.22	1.52	−50	+ .08	−1.13	0.47	1	− .24
23.....	5	.19	1.35	−37	+ .10	−0.83	0.62	2	− .09
Nov. 19.....	12	.11	1.53	−38	.00	−0.85	0.68	3	− .03
25.....	1	.....	1.77	+31	+ .07	−0.77	1.07	1	+ .36
Dec. 17.....	3	.04	1.85	−57	− .11	−1.30	0.44	1	− .27
26.....	2	0.03	1.56	+44	+0.14	−1.13	0.57	1	−0.14
					Weighted mean		0.71		

Table VIII gives the results obtained from Herschel's 19 nights of observation. The first column gives the date; the second, the number of stars compared with the moon; the third, the average deviation of the individual determinations of the moon's brightness from the mean; the fourth, the mean stellar magnitude of the moon's image at 100 inches distance; the fifth, the phase-angle, negative before the full; the sixth, the correction to mean distance; the seventh, the correction to full moon; the eighth, the deduced

magnitude of the image of the mean full moon; the ninth, the weight assigned, and the tenth, the residual from the final mean. The observations of the variable stars α Orionis and η Argus were excluded from these calculations, and also one observation each of α Bootis, β Centauri, and α Lyrae, for reasons given by Herschel.

Some sort of "night error," affecting all the observations of one night in the same direction, is evidently present. Weights have therefore been assigned which are roughly proportional to the square root of the number of observations. The probable error of the unit of weight, computed from the residuals, is $\pm 0^{M}.17$, which is not large, in view of the character of the observations.

Herschel's observations thus indicate that the image of the mean full moon, as seen in his astrometer at a distance of 100 inches, was of magnitude 0.71 ± 0.03 on the Harvard scale. To find the magnitude of the moon itself, it is necessary to correct for the influence of the small lens, and for the loss of light in passing through the lens and prism. Herschel states that the focal length of the lens was 0.2253 inch. He gives no details regarding the way in which this was determined, but it is probable that this precise statement by so careful an observer represents the result of accurate measurements. This would make the moon brighter than its image in the ratio $(100)^2:(0.2253)^2$, or by 13.24 magnitudes, to which must be added the instrumental losses. The latter can be estimated from Herschel's data. The prism was of crown glass (by Fraunhofer, and of fine workmanship) and of refractive index 1.571. By Fresnel's formula,[1] the loss of light by reflection from the surface at perpendicular incidence should be $0^{M}.055$. Assuming the lens to be similar to the prism, the total loss by reflection at the four surfaces would be $0^{M}.22$. Herschel does not give the dimensions of the prism, but from his figure of the astrometer, which appears to be drawn to scale, it would seem to have been about one inch on the rectangular faces. As the lens was only 0.12 inch in diameter, the whole thickness of glass traversed by the light was probably about an inch. Now, according to Vogel,[2] the transmission of light through 100 mm of ordinary crown glass is 0.85

[1] Preston, *Theory of Light*, pp. 346–347.

[2] *Astrophysical Journal*, **5**, 80, 1897.

for the visual rays. The loss by absorption in Herschel's apparatus may therefore be estimated at $0^{M}.04$, and the whole instrumental loss as $0^{M}.26$. The value of the stellar magnitude of the mean full moon resulting from his observations is therefore -12.79 on the Harvard scale.

This makes the moon $12^{M}.85$, or 138,000 times brighter than α Centauri. Herschel[1] himself, from his comparisons with this star alone, deduced the very different ratio of 27,400. The discrepancy is, however, easily explained. In the first place, Herschel reduced his observations to full phase by means of Euler's formula, which gives far too small a correction. The observations near full, taken separately, according to his statements, give a ratio of 45,000, and if the true corrections for phase had been applied, this would have been over 50,000. Secondly, Herschel expressly states, in the footnote in which he explains his method of reduction, that the quantity called M in his original memoir, and defined by equation (1) above, is "500 times the actual illuminating power of the moon at the time of observation, that of the mean full moon being unity." But, according to equation (1), M is 1000 for the mean full moon, and Herschel's numerical calculations bear this out. There seems, therefore, no escape from the conclusion that, in this later addition to his work, Herschel inadvertently confused the brightness of the mean full moon and the mean brightness of the moon at all phases, which, according to Euler's formula, is just half as great, and that his numerical result must be doubled to give the value of the former. The introduction of these two corrections would raise Herschel's value of the ratio to about 110,000. The outstanding difference is probably due to differences in the methods of reduction, and in the estimates of the instrumental losses.

b) ***Bond's observations.***—These consist of two independent series of comparisons, one with Venus and Jupiter, the other with the sun. In the first,[2] the images of the planets, reflected from a silvered glass globe, were compared with those of a standard lamp at adjustable distances, exactly as in the observations on the moon, and often on the same nights. From observations on 6 nights

[1] *Outlines of Astronomy* (New York, ed. 1876), p. 595.

[2] *Memoirs of the American Academy of Arts and Sciences*, N.S., **8**, 250, 1861.

he concludes that Venus, at phase-angle 68°.8, and reduced to mean distance from the sun and unit distance from the earth, was $8^{M}.08$ fainter than his lamp at a distance of one foot, and from 4 nights' observations (rejecting one night for good cause) he concludes that Jupiter at mean opposition was $9^{M}.51$ fainter than the same standard. The average residual for a night was $\pm 0^{M}.14$ for Venus and $\pm 0^{M}.12$ for Jupiter. Bond's assumed value for the light of the mean full moon is $0^{M}.06$ brighter than the lamp at one foot, and the zero correction given in Table V means that, if reduced anew with the light-curve of Table III, this would be $0^{M}.24$ brighter still. Hence Bond's observations make the mean full moon $8^{M}.38$ brighter than Venus, and $9^{M}.81$ brighter than Jupiter at the given phases. Müller's magnitudes for the planets at these times, reduced to the Harvard scale, are -3.71 and -2.29; so that the moon's magnitude comes out -12.09 from the comparisons with Venus, and -12.10 from those with Jupiter.

In spite of the agreement of the two, this result deserves little weight. Bond did not employ a blue screen to make the light of his lamp similar in color to that of the moon or of the planets, and he states that the lamp was decidedly red in comparison with the moon. The same reflecting globe was used in observing the moon and the planets, and the image of the lamp appeared to the eye more than a thousand times fainter in the second case than in the first. The Purkinje effect would therefore come into play to a high degree, and would operate to make the lamp appear relatively faint at the lower intensity, and hence to make the measured difference in magnitude between the moon and the planets too small, as appears from comparison with other observations to be actually the case.

Bond's comparisons of the moon and sun[1] were made by a very similar method. The light of the sun, reflected from a silvered globe, was again reflected in a second, and compared with the reflection of a "Bengola light" in the latter, and the moon was subsequently observed in exactly the same way. The obvious weakness of the method, severely criticized by Zöllner,[2] is the uncertainty whether the light of these fireworks could be considered as

[1] *Ibid.*, p. 287, 1861.

[2] *Photometrische Untersuchungen*, p. 116.

even roughly constant, even if "those of the same size and manufacture were used throughout, and a fresh one burned for each comparison." Some idea of the magnitude of the errors involved can, however, be derived from the fact that the five observations of the sun, on two days, show an average deviation from the mean ratio of sunlight to Bengola light of $\pm 0^{M}.20$, and the six of the moon (all on one night, when its phase was 7°.6 and its altitude 24°) a deviation of $\pm 0^{M}.18$. From these comparisons Bond concludes that the sun is 471,000 times, or $14^{M}.18$, brighter than the mean full moon. The correction to full moon from the observed phase is $0^{M}.03$ according to Bond's curve, and $0^{M}.15$ by Table III, so that the difference of magnitude, on the system of the present paper, is 14.06, which, with the value of the sun's magnitude already found, makes that of the full moon −12.66. The probable error of this value, deduced from the observations, is $\pm 0^{M}.12$; but it may really be considerably greater.

c) *Zöllner's observations.*[1]—These fall again into two groups. In those made by his "second method" a point image of the moon was compared with an artificial star, and this again with Capella (using a different optical train). Eight nights on the moon and 11 on the star (separated by an average interval of some three months) make the difference in magnitude between the star and the mean full moon $12^{M}.39$ when reduced with Zöllner's formula. Applying the zero correction found above, this becomes $12^{M}.38$ and the moon's magnitude −12.17 on the Harvard scale.

By his "first method" the moon and sun were compared with a standard lamp, with a polarizing photometer of different type. Ten days' observations on the sun, and 6 nights' on the moon, when reduced with the light-curve here adopted, give for the difference of magnitude $14^{M}.45$, and for the magnitude of the moon −12.27.

d) *The observations of W. H. Pickering.*—These observations, which have already been described,[2] were referred to the stars by removing the pentane lamp, with blue screen as used in the measures of moonlight, to such a distance that it appeared of the same brightness as Arcturus (which seemed nearly of the same color). Observations were unfortunately made on only 4 nights, and their

[1] *Photometrische Untersuchungen*, pp. 90–115, 124.

[2] *Harvard Annals*, **61**, 63–68.

average deviation from the mean was $\pm 0^{M}.24$, so that the probable error of the determination of the relative brightness of the star and lamp from the mean of the four is $\pm 0^{M}.12$, while that of the relative brightness of the lamp and the mean full moon (reducing the 24 comparisons by means of Table III) is only $\pm 0^{M}.024$. Pickering finds that the magnitude of the latter is -12.50, which becomes -12.51 on applying the zero correction.

e) From the comparisons of sunlight and moonlight with the standard candle (see below) the ratio of the two is found to be 428,000, corresponding to a difference of magnitude of 14.08.

Rejecting Bond's comparisons with the planets, for reasons already stated, the mean results of each observer for the magnitude of the mean full moon are as follows:

		Residual
Herschel	−12.79	−0.24
Bond	−12.66	− .11
Zöllner	−12.22	+ .33
Pickering	−12.51	+ .04
Comparison with candle	−12.64	−0.09

If Bond's determination is given half the weight of the others, the general mean is -12.55, with a probable error of ± 0.07, as computed from the residuals. The actual uncertainty of this result is probably greater, as some of the determinations may be affected by systematic errors.

From the values here found for the magnitudes of the sun and moon, it appears that the sun exceeds the mean full moon in brightness by 14.17 magnitudes, or 465,000 times, with an uncertainty of fully 10 per cent.

f) The photographic magnitude of the moon is well determined by King's long series of observations. The value $-11^{M}.20$, which he originally gave, requires a correction of $+0^{M}.07$ to reduce it to his definitive system of magnitudes,[1] and a further one of $-0^{M}.24$ for the difference between his light-curve and that of Table III, making the final value $-11^{M}.37$. This makes the color-index of moonlight $+1^{M}.18$, about that of a star of Class K0, and $0^{M}.39$ greater than that of the sun. At first glance it might be doubted

[1] *Ibid.*, **59**, 153.

whether the moon is really as red as this; but the spectrophotometric observations of Wilsing and Scheiner[1] proved that her surface is strongly colored. Their measurements, made on a bright and a dark region, showed sensibly the same behavior for both, the relative reflecting power increasing from 0.61 at λ 4480 to 0.64 at λ 5130, 0.87 at λ 5840, and 1.00 at λ 6380. This indicates a reflecting power about 20 per cent less in the photographic than in the visual region, and hence a color-index about $0^{M}.25$ greater than that of the sun, in fair agreement with the results of the photometric measures.

V. THE STELLAR MAGNITUDE OF THE STANDARD CANDLE

Though this quantity is not directly related to the question of albedo, it may appropriately be discussed here.

a) *Sunlight and terrestrial standards.*—The older comparisons,[2] summarized by Müller, must have been systematically in error, for their results are all much too low. The first approximately correct value appears to have been Fabry's, already quoted, which made the sun in the zenith equal to 100,000 meter-candles. Much the most extensive and precise series of observations appears to be the very recent one by Kimball,[3] at Mount Weather, Virginia. With a photometer of precision, and carefully tested standards of light, he measured the intensity of illumination of a horizontal surface by the combined light of the sun and sky, and also by the sky when direct sunlight was cut off, and thus deduced the intensity of normal illumination by the latter over a large range of zenith distances on many days.

From the data given for various air-masses in his Table IV, it is possible on many days to extrapolate to the zenith (air-mass unity) and determine the transmission of light by the atmosphere on each day. If the results are grouped according to the brightness of the light of the sky when the sun was hidden, that is, according to the haziness of the weather, the following mean values are

[1] *Publikationen des astrophysikalischen Observatoriums zu Potsdam*, **20**, No. 61, 1909.

[2] *Photometrie der Gestirne*, pp. 309–312.

[3] *Monthly Weather Review*, **43**, 650, 1914.

obtained, the intensity of the light being given in foot-candles, which units Kimball uses throughout:

Days	Light of Sky	Zenith Sun	Atmospheric Transmission
5	880	9750	0.782
5	1035	9550	0.780
5	1155	9550	0.736
3	1420	8650	0.675
4	2400	6700	0.677

As might be expected, the brightness of direct sunlight falls off seriously in hazy weather. For comparison with other results of astronomical photometry, it is probably desirable to take only the observations made in fairly clear weather, the first three groups in the table. Observations on seven more clear days (with sky light at noon less than 1200 foot-candles) can also be employed by reducing them to the zenith with the mean transmission 0.766 found for the three groups above.

The mean of the 22 days' observations in clear weather gives, for the intensity of sunlight from the zenith, 9600 foot-candles, or 103,000 meter-candles, with an average deviation for the individual determinations of ±5.4 per cent. According to Kimball's statements, the probable error of the mean, including systematic errors, should not exceed 5 per cent.

This determination makes the zenith sun 12.53 magnitudes brighter than a standard candle at 1 m distance. With the atmospheric transmission found above, the sun outside our atmosphere would be $0^{M}.29$ brighter, and its light equal to 134,500 meter-candles.

b) ***Moonlight and standard candle.***—The latest and probably the best determination of the intensity of moonlight compared with artificial standards is that by Graff,[1] who compared, with a Weber photometer, the light of the moon with that of an amyl-acetate lamp shining through pale-blue glass, and this with a Hefner lamp. Observations on 5 nights gave for the light of the mean full moon in the zenith 0.269±0.014 Hefner-meter-candles. The light-curve used in reducing the observations, all of which were made

[1] *Astronomische Nachrichten*, **198**, 14, 1914.

within 24° of the full, to the latter phase was so nearly identical with that of Table III that no correction is required. Graff also states that the mean of a considerable number of older determinations, most of which are satisfactorily accordant, is 0.251 English normal candle, or 0.267 Hefner candle, at 1 m. The ratio of moonlight to the meter-candle appears therefore to be very well determined. Since the Hefner candle is equal to 0.90 international standard candle, full moonlight may be taken as 0.241 meter-candle on this scale, and the international meter-candle as 1.55 magnitudes brighter than the mean full moon.

c) *Standard candle and starlight.*—Direct comparisons of laboratory standards of light with the stars are complicated by the great difference in color, and depend upon the accuracy with which the transmission of the blue screens which are necessary can be determined. Fabry's result,[1] that a *bougie décimale* at a distance of 780 m would appear as bright as Vega, appears to be the best available direct comparison, as he determined the transmission of his screen from the mean of a large number of laboratory measurements. This makes the meter-candle $14^{M}.46$ brighter than Vega.

W. H. Pickering's result,[2] that a standard candle at 1 m is of magnitude −13.60 on the Harvard scale, depends on a transmission coefficient determined from comparisons of moonlight with the lamp, with and without the blue screen, on but a single night and is necessarily of little weight, because of the small number of comparisons, and especially since the intensity of the illumination of the screen of the shadow photometer with which the observations were made—that of moonlight—was so low that the Purkinje phenomenon is to be feared. It would operate to make the measured brightness of the unscreened lamp lower, and the transmission of the screen higher, than would have been found had the observations been made with illumination of sufficient intensity to lie outside the dangerous region, and so to give too low a value for the brightness of the standard candle. As the result is actually much lower than that of any other observer, it may be excluded from the mean, for the reasons stated.

[1] *Comptes Rendus*, **137**, 1242, 1903.

[2] *Harvard Annals*, **61**, 69, 1908.

The other determinations give the following values for the stellar magnitude of the standard candle at a distance of 1 m.

		Weight
Kimball (sun)	−14.19	3
Graff and others (moon)	−14.09	2
Fabry (Vega)	−14.32	1

With the weights assigned, the mean is −14.18, with an apparent probable error of about $\pm 0^{M}.05$.

Photographically, E. S. King has found[1] that a standard 2-candle electric lamp, supplied by the Bureau of Standards, was of magnitude −12.06 at a distance of 1 m. The color-index of this lamp was therefore $+2^{M}.87$, so that, compared with stars of Class A, the lamp was 14 times fainter photographically than visually. Other standards of light might, however, have very different color-indices.

It follows from the foregoing that a light of the same mean color as that of the stars, and of intensity which, measured under high illumination, was of one candle-power, would appear to be of stellar magnitude 0.82 at a distance of 1 km, and 1.85 at a distance of a mile. An actual standard candle, of the ordinary varieties, if removed to these distances, would undoubtedly appear much fainter, on account of the Purkinje effect. Pickering's measures indicate that the difference would probably exceed half a magnitude.

It is evident from the foregoing summary that the precise determination of the relative brightness of the full moon and of the stars, and the direct comparison of the stars and terrestrial standards of light, by trustworthy visual and photo-visual methods, are urgently desirable. No direct visual measurements of the ratio of sunlight to moonlight, and but one of that of moonlight to starlight, appear to have been made in the past fifty years. Most of the existing determinations are derived from incidental observations by astronomers whose main attention was directed elsewhere. A determination of these fundamental photometric constants with modern precision would well repay the undivided attention of a competent observer for a considerable time.

PRINCETON UNIVERSITY OBSERVATORY
January 20, 1916

[1] *Ibid.*, **59**, 275.

PHOTOMETRIC STUDY OF THE ECLIPSING VARIABLE RV OPHIUCHI

By RAYMOND S. DUGAN

The variability of this star (B.D.+7°3404) was discovered by Mrs. Fleming,[1] and independently by Mme Ceraski,[2] in comparing photographic plates. It has been further observed by Blajko,[2] Hartwig,[3] and Wendell.[4] In 1906, after quite a long watch, I finally observed the star entering eclipse on April 18. In May, I was able to announce provisional elements, which agreed with those given by Wendell at the same time.[5] The length of the period, the position of the star, and many interruptions delayed the completion of the observation of the light-curve until 1914. The position of the star is (1900) $\alpha = 17^h29^m8$, $\delta = +7°19'$, and its provisional designation as a variable was 136.1904.

The 14,128 measures were made with the polarizing photometer in the usual manner and reduced in duplicate.[6] The tables in *Harvard Annals*, **33**, have been extended to three decimal places. Atmospheric absorption was allowed for and the times reduced to the sun. The usual comparison star was a=B.D.+7°3403. A second star, b=B.D.+7°3405, was also used for comparison on a few nights.

From an inspection of the observations by Wendell and by myself of eclipses scattered over 994 periods I assumed the preliminary elements

J.D. 2420269.70597+3^d.687126 E. Elements 1.

With these approximate elements all my observations taken during primary minima were collected and formed into a mean curve.

[1] *Harvard Circular*, No. 80.

[2] *Astronomische Nachrichten*, **166**, 155, 1904.

[3] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **40**, 329, 1905.

[4] *Harvard Annals*, **69**.

[5] *Harvard Bulletin*, No. 244.

[6] *Contributions of Princeton Observatory*, No. 1.

This was drawn on transparent paper and used to determine the times of minima from the individual plots of minima observed by Wendell and by myself. In Table I, the fourth column contains these observed times. The weights are assigned in accordance with the accuracy with which the observations, from their number and distribution, apparently determine the time of mid-eclipse.

The residuals $O.-C._1$, when plotted, indicate, with considerable clearness, the need of a sine term of small amplitude. One-half the amplitude is apparently about 2 minutes, the period about 1280 E, while the residual becomes zero at about -125 E. The following new elements were, therefore, assumed:

J.D. 2419808.81513$+3^d.687126$ E$-0^d.0014$ sin $0^\circ.28$ E. Elements 2.

A least-squares solution was made, regarding the amplitude and phase of the sine term as known. The resulting corrections were

$$dT = -0^m.40 \pm 0^m.37$$
$$dP = +0^m.001 \pm 0^m.0011$$
$$dq = -0^\circ.06 \pm 0^\circ.036$$

and the residuals are given in the last column of Table I. The sum of the weighted squares of residuals was reduced from 104.62 from elements 1, to 43.46 from elements 2, to 36.01 from the final elements. The final elements are

J.D. 2419808.81485$+3^d.6871267$ E$-0^d.0014$ sin $0^\circ.22$ E$=$1913 Feb. $9^d19^h33^m.4$ G.H.M.T.$+3^d16^h29^m27^s.75$ E-2^m sin $0^\circ.22$ E. Elements 3.

Elements and observed times appearing in this paper are heliocentric.

TABLE I

Observer	Epoch	J.D.	G.H.M.T. Observed	No. of Obs.	Wt.	$O.-C._1$	$O.-C._2$	$O.-C._3$
Wen.....	−994	2416604	16^h50^m	5	2	$-1^m.8$	$0^m.0$	$+0^m.5$
Wen.....	−984	16641	13 44	3	1	−2.3	−0.6	−0.2
D.....	−796	17334	18 4.5	13	6	−0.8	−0.5	−0.8
D.....	−585	18112	17 44	17	6	+2.7	+1.1	+1.4
Wen.....	−578	18138	13 9.5	4	1	+1.7	+0.1	+0.4
D.....	−486	18477	18 19.5	23	10	+1.2	−0.8	−0.2
D.....	−390	18831	17 26.5	6	1	−0.3	−2.2	−1.3
D.....	−281	19233	14 55	4	1	−2.8	−4.2	−3.4
D.....	−112	19856	17 59	2	1	+2.2	+2.3	+2.7
D.....	0	20269	16 55	17	9	−1.3	−0.2	−0.1

The individual observations were first weighted according to observing conditions. Normals were then formed and plotted, and a curve drawn through them. Residuals were then read off. The observations were corrected for the average residual (usually of the whole night, but sometimes, where there was an abrupt change, for a part of the night). The weights of the observations were then modified according to the magnitude of the average remaining residual. These remaining residuals were found to have the characteristics of accidental errors. Normals, usually of five observations each, were formed and the sum of the weights diminished in cases where all the observations entering into a normal were not made on different nights, the weight of a normal made up of five observations on one night suffering the greatest reduction. The current number of the normal, the current number of the supernormal into which the normals are combined for the final solution, the phase, the weight, the observed difference in magnitude between the variable and comparison star *a*, are given in the first five columns of Table II.

The mean light-curve of RV Ophiuchi, while presenting the relatively simple conditions of a circular orbit and a total eclipse, is decidedly puzzling. The observations, which I have no reason to think are inferior to those of RT Persei and Z Draconis, bring out clearly a strong asymmetry combined with an apparently non-periodic succession of hollows and humps. The observed secondary minimum is barely deep enough to satisfy the representation of primary minimum on the assumption of uniformly brilliant disks. Any darkening toward the limb requires a still deeper secondary minimum. As the depth of secondary minimum in a light-curve of this form should be strongly affected by darkening, hopes had been entertained of even determining the degree of darkening in this case. Although, as will be seen, the effect of mutual radiation is about the same as in the systems RT Persei and Z Draconis, the ellipticity is much greater than in the two other cases. This was again surprising, since the two stars are so far apart as to warrant no expectation that an ellipticity effect could be detected.

As the depth (*d*) of secondary minimum is comparable with the ellipticity (*c*) and effects of mutual radiation (*b*) it seemed wise to

TABLE II

No.	Super-normal No.	Phase	Wt.	$a-v$	O.−C.	O.−C.′
1.....	38	−6^{h}49^m.5	14	+0^M.991	−0^M.005	+0^M.003
2.....	38	28.0	14	.973	− .022	− .015
3.....	38	6 5.3	13	.985	− .009	− .002
4.....	38	5 50.6	9	1.001	+ .008	+ .015
5.....	38	28.4	6	.991	− .002	+ .004
6.....	38	5 10.1	6	.969	− .023	− .017
7.....	38	4 50.3	6	1.008	+ .017	+ .023
8.....	38	4 29.0	3	1.003	+ .017	+ .022
9.....	16	3 53.7	5	.866	− .034	− .027
10.....	15	37.0	9	.819	− .013	− .005
11.....	14	21.0	7	.708	− .052	− .043
12.....	13	3 5.2	10	.669	− .001	+ .010
13.....	12	2 54.7	10	.553	− .053	− .040
14.....	11	40.6	8	.467	− .038	− .024
15.....	10	31.6	11	.367	− .065	− .051
16.....	9	18.4	12	.265	− .062	− .048
17.....	9	2 8.3	8	+0.243	+ .013	+ .026
18.....	8	1 58.4	11	+0.135	+ .007	+ .020
19.....	7	44.8	13	−0.031	.000	+ .011
20.....	6	29.5	12	−0.224	.000	+ .010
21.....	5	17.8	13	.414	− .030	− .022
22.....	4	1 5.5	11	.603	− .040	− .034
23.....	3	0 54.7	11	.745	− .030	− .025
24.....	2	39.3	13	.924	− .034	− .031
25.....	1^I	26.0	13	1.001	− .021	− .019
26.....	1	14.5	11	.994	+ .004	+ .005
27.....	1	− 0 0.3	11	1.002	− .004	− .004
28.....	1	+ 0 15.8	10	.973	+ .025	+ .024
29.....	1^I	28.2	11	.984	− .014	− .016
30.....	.....	41.4	12	.979	− .109	− .112
31.....	3	0 53.9	14	.766	− .040	− .045
32.....	4	1 7.4	15	.568	− .030	− .036
33.....	5	17.0	15	.393	+ .032	+ .024
34.....	6	28.4	11	−0.235	− .005	− .015
35.....	7	39.1	9	−0.074	+ .024	+ .013
36.....	7	49.4	12	+0.054	+ .026	+ .014
37.....	8	1 59.3	12	+0.164	+ .027	+ .014
38.....	9	2 10.6	9	.281	+ .031	+ .018
39.....	10	21.2	12	.359	+ .009	− .005
40.....	11	35.3	8	.505	+ .043	+ .029
41.....	12	45.6	14	.553	+ .008	− .006
42.....	13	2 57.3	13	.649	+ .024	+ .012
43.....	14	3 9.9	14	.736	+ .036	+ .025
44.....	15	3 34.0	11	.853	+ .031	+ .023
45.....	17	+ 4 8.1	12	+0.964	+0.020	+0.014

TABLE II—*Continued*

No.	Super-normal No.	Phase	Wt.	$a-v$	O.−C.	O.−C.′
46.....	19	+4^{h}35^{m}6	9	+0^M.993	+0^M.003	−0^M.002
47.....	19	5 5.6	8	.990	− .002	− .008
48.....	19	27.4	8	1.018	+ .025	+ .019
49.....	19	5 47.5	9	.999	+ .005	− .002
50.....	19	6 3.7	8	1.005	+ .010	+ .003
51.....	19	25.3	8	.985	− .010	− .017
52.....	19	6 52.7	6	1.014	+ .018	+ .010
53.....	19	7 17.4	3	.970	− .028	− .036
54.....	19	7 41.2	6	1.008	+ .009	.000
55.....	19	8 22.2	5	.974	− .027	− .036
56.....	19	9 0.3	5	.994	− .009	− .019
57.....	19	9 30.7	5	.987	− .017	− .027
58.....	20	10 17.2	8	.998	− .008	− .019
59.....	20	12 27.8	7	.966	− .046	− .059
60.....	20	13 53.0	10	.984	− .032	− .046
61.....	20	14 21.5	9	.990	− .027	− .041
62.....	20	14 38.4	12	1.010	− .008	− .022
63.....	21	17 40.9	10	1.054	+ .028	+ .012
64.....	21	18 30.5	12	1.043	+ .015	− .001
65.....	21	18 47.8	10	1.068	+ .040	+ .024
66.....	21	19 4.2	10	1.057	+ .028	+ .012
67.....	21	23.0	7	1.059	+ .029	+ .013
68.....	21	19 41.1	8	1.046	+ .016	.000
69.....	21	20 2.7	6	1.050	+ .020	+ .004
70.....	21	27.0	9	1.051	+ .020	+ .004
71.....	22	42.7	8	1.047	+ .015	− .001
72.....	22	20 58.8	9	1.070	+ .040	+ .023
73.....	22	21 15.8	8	1.032	.000	− .017
74.....	22	23 18.9	11	1.074	+ .040	+ .024
75.....	22	24 20.6	7	1.034	.000	− .016
76.....	23	27 25.0	5	1.047	+ .017	+ .002
77.....	23	28 14.7	5	1.069	+ .040	+ .025
78.....	23	28 57.5	6	1.066	+ .038	+ .023
79.....	23	30 2.8	6	1.041	+ .015	+ .001
80.....	23	31 12.4	6	1.047	+ .023	+ .010
81.....	24	32 30.9	6	1.034	+ .013	+ .001
82.....	24	33 42.5	4	1.014	− .004	− .015
83.....	24	34 32.4	6	1.051	+ .034	+ .023
84.....	24	35 28.4	6	1.035	+ .020	+ .010
85.....	24	35 58.2	6	1.044	+ .030	+ .021
86.....	24	36 33.1	6	1.029	+ .016	+ .007
87.....	25	37 44.6	8	.991	− .020	− .028
88.....	25	38 13.7	10	1.018	+ .008	+ .001
89.....	25	38 39.5	8	.994	− .014	− .021
90.....	25	+39 21.4	7	+1.034	+0.027	+0.021

TABLE II—*Continued*

No.	Super-normal No.	Phase	Wt.	a−v	O.−C.	O.−C.′
91.....	26	+39^{h}59^{m}9	7	+0^M.978	−0^M.028	−0^M.033
92.....	26	40 30.0	5	+1.013	+ .017	+ .013
93.....	27	41 48.6	6	.973	+ .006	+ .003
94.....	28	42 14.5	8	.942	− .013	− .015
95.....	28	31.7	9	.947	.000	− .002
96.....	29	42 52.1	8	.946	+ .008	+ .006
97.....	29	43 6.2	6	.938	+ .004	+ .003
98.....	30	22.1	8	.917	− .013	− .014
99.....	30	35.9	8	.943	+ .015	+ .014
100.....	31	43 45.8	8	.925	− .002	− .003
101.....	31	43 56.4	8	.885	− .042	− .042
102.....	31′	44 6.5	10	.938	+ .011	+ .011
103.....	31′	18.3	9	.960	+ .033	+ .033
104.....	31′	27.1	6	.922	− .005	− .005
105.....	31	36.3	12	.987	+ .060	+ .060
106.....	30	46.6	9	.947	+ .019	+ .020
107.....	30	44 57.5	8	.941	+ .012	+ .013
108.....	29	45 11.2	9	.973	+ .041	+ .042
109.....	29	25.2	7	.941	+ .006	+ .007
110.....	29	45 42.5	8	.947	+ .005	+ .007
111.....	28	46 20.0	7	.949	− .008	− .005
112.....	27	47 8.9	12	.970	− .008	− .005
113.....	27	28.9	10	.986	− .003	+ .001
114.....	27	47 46.7	10	.993	.000	+ .004
115.....	26	48 16.3	8	1.005	+ .004	+ .009
116.....	32	+48 37.2	9	1.014	+ .009	+ .014
117.....	32	−39 31.1	7	1.005	.000	+ .006
118.....	32	11.0	13	.961	− .047	− .041
119.....	32	39 0.3	12	1.030	+ .022	+ .028
120.....	32	38 51.8	11	1.003	− .005	+ .001
121.....	32	43.0	13	.992	− .017	− .011
122.....	32	32.6	13	1.037	+ .028	+ .035
123.....	32	38 13.1	12	.993	− .016	− .009
124.....	32	37 49.6	13	1.036	+ .026	+ .033
125.....	32	32.9	14	.987	− .023	− .015
126.....	32	20.8	14	1.020	+ .009	+ .017
127.....	32	37 4.7	13	1.018	+ .006	+ .014
128.....	32	36 43.9	10	.990	− .022	− .014
129.....	32	36 22.2	9	1.018	+ .005	+ .014
130.....	32	35 55.3	10	.972	− .042	− .033
131.....	32	42.0	7	1.015	.000	+ .009
132.....	32	35 6.7	2	1.000	− .016	− .006
133.....	33	33 49.8	5	.985	− .034	− .023
134.....	33	32 34.5	5	.982	− .040	− .028
135.....	33	−31 32.4	6	+0.971	−0.052	−0.039

TABLE II—*Continued*

No.	Super-normal No.	Phase	Wt.	$a-v$	O.−C.	O.−C.′
136.....	33	−30^{h}48^{m}8	7	+1^{M}005	−0^{M}020	−0^{M}006
137.....	33	30 19.5	7	.980	− .045	− .031
138.....	33	29 51.6	8	.989	− .037	− .023
139.....	34	29 21.6	8	.974	− .052	− .038
140.....	34	28 56.2	9	1.020	− .008	+ .007
141.....	34	28 20.1	11	1.018	− .011	+ .004
142.....	34	27 39.6	8	1.005	− .025	− .010
143.....	34	27 14.9	8	1.016	− .015	.000
144.....	34	26 57.0	9	1.000	− .032	− .017
145.....	34	38.5	8	.988	− .044	− .028
146.....	34	26 19.3	8	.998	− .034	− .018
147.....	34	25 47.0	9	1.005	− .027	− .011
148.....	34	25 11.2	4	1.042	+ .010	+ .026
149.....	35	24 23.2	6	1.037	+ .005	+ .021
150.....	35	23 33.7	6	.993	− .039	− .023
151.....	35	22 48.0	6	1.016	− .016	.000
152.....	35	22 15.8	6	1.033	.000	+ .017
153.....	35	21 48.6	6	1.000	− .032	− .015
154.....	35	27.7	5	.982	− .050	− .033
155.....	35	21 8.2	6	.996	− .036	− .019
156.....	35	20 51.4	6	1.054	+ .022	+ .038
157.....	35	34.2	5	1.050	+ .020	+ .036
158.....	35	20 15.9	4	1.055	+ .024	+ .040
159.....	35	19 56.6	3	1.030	.000	+ .016
160.....	35	31.5	7	1.009	− .020	− .004
161.....	36	19 8.9	6	1.060	+ .030	+ .046
162.....	36	18 51.1	10	1.037	+ .009	+ .025
163.....	36	27.9	9	1.026	.000	+ .016
164.....	36	18 1.2	8	1.042	+ .015	+ .031
165.....	36	17 36.0	8	1.001	− .025	− .009
166.....	36	17 11.6	8	1.044	+ .019	+ .034
167.....	36	16 40.8	4	1.036	+ .012	+ .027
168.....	36	15 29.6	4	1.043	+ .023	+ .038
169.....	37	13 59.0	2	1.020	+ .004	+ .018
170.....	37	11 31.9	10	1.028	+ .019	+ .031
171.....	37	10 36.9	15	1.049	+ .043	+ .055
172.....	38	9 22.4	13	.990	− .013	− .002
173.....	38	8 58.0	12	.988	− .013	− .002
174.....	38	36.9	12	1.016	+ .016	+ .025
175.....	38	18.6	16	1.011	+ .012	+ .021
176.....	38	8 3.3	14	.988	− .010	− .001
177.....	38	7 51.1	16	.996	− .002	+ .007
178.....	38	32.5	11	.999	+ .002	+ .010
179.....	38	−7 11.6	14	+0.981	−0.015	−0.007

determine these quantities, as well as the light (a) of the system at longitude 90° and the time of secondary mid-eclipse, from the whole curve outside of primary minimum.[1] All these observations were combined into 20 supernormals (this solution was made before the observations were corrected for night errors and the weights modified). The second-order terms were computed with the assumed values of the unknowns. The result of the solution was

$$a = 1.021 \pm 0.002$$
$$b = 0.007 \pm 0.004$$
$$c = 0.042 \pm 0.004$$
$$d = 0.052 \pm 0.010$$
$$\text{Time of sec.} = \text{time of primary} + \tfrac{1}{2}P + 0^{m}.6 \pm 20^{m},$$

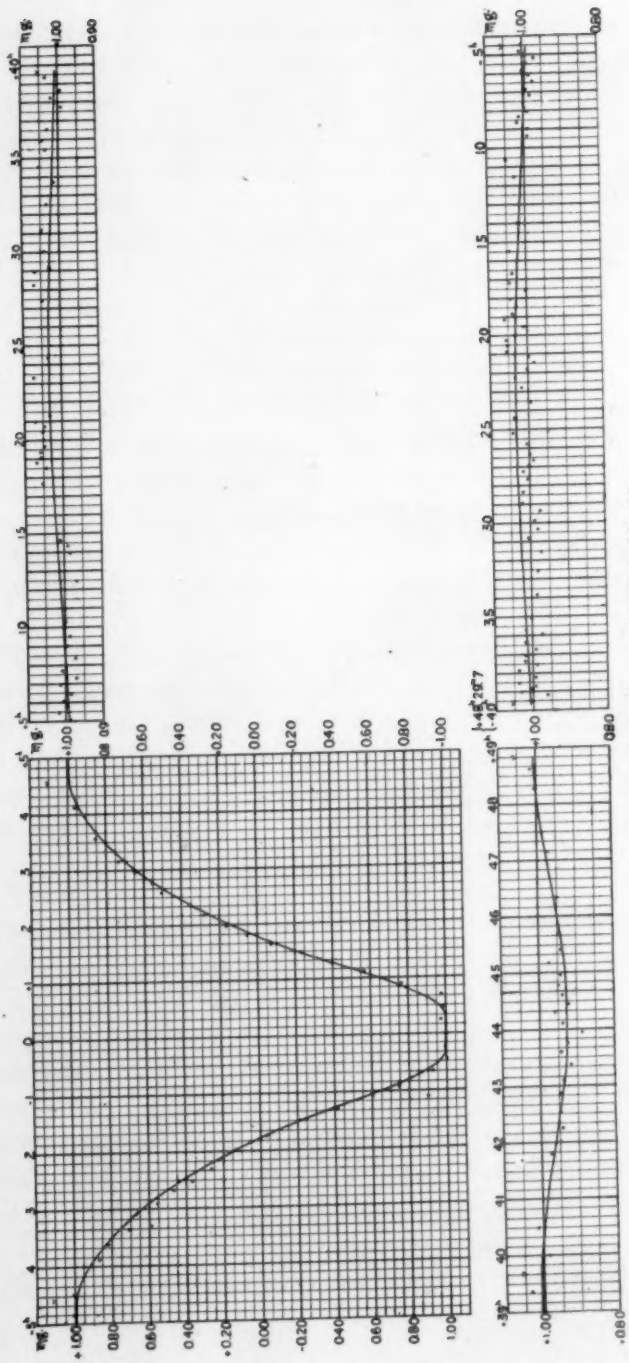
the unit of intensity being the average brightness between eclipses, $a - v = 1^{M}.013$. The probable error of weight unity is ± 0.020, which is, on the average, that of three normals. The probable error of a single normal would be, therefore, ± 0.034. This is not surprisingly large in view of the pronounced asymmetrical difference between the two branches outside of eclipse.

The curve was then rectified to intensity 1.0288, corresponding to $a - v = 1^{M}.045$. From the length of the constant phase during primary minimum we know that this eclipse is total. From the depths of the two minima we find that the radius of the brighter star is ($k =$) 0.54 that of the fainter. In the following, Russell's methods and notation are used.[2] With this value of k no values of $A = \sin^2\theta_2$ and $B' = \dfrac{\sin^2\theta_2 - \sin^2\theta_3}{1 - 2c\cos^2\theta_3}$ could be found which gave a satisfying representation of the mean of the two branches of primary minimum. $k = 0.54$, $A = 0.0228$, $B' = 0.0175$ represents the ascending branch fairly well, but the whole eclipse calls for a value of k in the neighborhood of 0.64. This value of k requires a depth of secondary minimum exceeding the observed by $0^{M}.012$. Any assumption of darkening calls for a still deeper secondary. The amount of ellipticity, which was well determined by the least-squares solution, is unexpectedly large.[3] Representations of primary minimum were

[1] Russell, *Astrophysical Journal*, 39, 407, 1914.

[2] *Astrophysical Journal*, 35, 36, and 39.

[3] See Shapley's table, *Contributions of Princeton Observatory*, No. 3, p. 115.



Computed Light-Curve of RV Ophiuchi

attempted, assuming the amount of ellipticity anticipated from experience, and assuming no ellipticity at all. In neither case was there much improvement. At this point the discussion of systematic residuals was undertaken, and still the representation was not satisfactory. The thought comes to one that it may be futile to attempt to find elements to represent the mean of the two branches of such an asymmetrical curve as the one under discussion.[1]

Since the value of k is affected by the elements of rectification, it seemed best to attempt a least-squares solution of the entire curve in 7 unknowns, A, B, k, a, b, c, and λ_1 the observed intensity during totality. The intensity during the annular phase of secondary minimum is expressed as a function of k and λ_1. The somewhat complicated equations, based on the assumption of uniformly brilliant disks, are as follows:

I. $$\sin^2\theta = A + \psi \cdot B \cdot n,$$

where

$$n = \frac{1 - z\cos^2\theta}{1 - z\cos^2\theta_3}, \quad z = 2c.$$

I′. $$dA + \psi \cdot n \cdot dB + \psi \cdot B \cdot \frac{\delta n}{\delta c} \cdot dc + B \cdot n \cdot \frac{\delta \psi}{\delta k} \cdot dk + B \cdot n \cdot \frac{\delta \psi}{\delta a} \cdot da = 0.$$

In primary eclipse:

II. $$l = a - b \cdot \cos\theta - a(a + b_1 + b_2 - \lambda_{1r}) - l' \cdot c \cdot \cos^2\theta + bc\cos^3\theta - l' \cdot \frac{c^2}{2}\cos^4\theta$$

where

$$l' = a - a(a + b_1 + b_2 - \lambda_{1r}); \quad \lambda_{1r} = l_1' + b_1 + b_2; \quad l_1' = \frac{\lambda_1}{\sqrt{1 - z}} + b_1;$$

also

$$a + b_1 + b_2 - \lambda_{1r} = a - b - \lambda_1 - l_1' \cdot c + bc - l_1' \cdot \frac{c^2}{2}.$$

Subscript r means rectified (but expressed in same unit as the observed intensities), and 1 refers to conjunction during total eclipse.

II′. $$dl = (1 - a)da - (\cos\theta - a)db + a \cdot d\lambda_1 - (l'\cos^2\theta - l_1' \cdot a)dc - (a + b_1 + b_2 - \lambda_{1r})da.$$

[1] *Astrophysical Journal*, **35**, **36**, and **39**.

Eliminating da between equations II′ and I′, and setting for brevity

$$y=a+b_1+b_2-\lambda_{1r}$$

and

$$x=B\cdot n\cdot\frac{\delta\psi}{\delta a}$$

we obtain for primary eclipse

$$dl=\frac{y}{x}\cdot dA+\frac{y}{x}\psi\cdot n\cdot dB+y\frac{\frac{\delta\psi}{\delta k}}{\frac{\delta\psi}{\delta a}}dk+a\cdot d\lambda_1+(1-a)da-(\cos\theta-a)db$$

$$-\left(l'\cos^2\theta-l_1'\cdot a+\frac{y}{x}\cdot\psi\cdot n\cdot\frac{\delta n}{\delta c}\right)dc.$$

During secondary we have

III. $$l=a-b\cos\theta-k^2a\lambda_{1r}-l'c\cos^2\theta+bc\cos^3\theta-l'\frac{c^2}{2}\cos^4\theta$$

where

$$l'=a-k^2a\cdot\lambda_{1r}$$

and

III′. $$dl=da-(\cos\theta+2k^2a)db-k^2a\cdot d\lambda_1-(l'\cos^2\theta-k^2al_1')dc-2ka\lambda_{1r}\cdot dk-k^2\lambda_{1r}\cdot da$$

and

$$dl=\frac{k^2\lambda_{1r}}{x}dA+\frac{k^2\lambda_{1r}}{x}\psi\cdot n\cdot dB+\left[k^2\lambda_{1r}\cdot\frac{\frac{\delta\psi}{\delta k}}{\frac{\delta\psi}{\delta a}}-2ka\lambda_{1r}\right]dk-k^2a\cdot d\lambda_1+da$$

$$-(\cos\theta+2k^2a)db-\left(l'\cos^2\theta+k^2al_1'+\frac{k^2\lambda_{1r}}{x}\cdot\psi\cdot n\cdot\frac{\delta n}{\delta c}\right)dc.$$

Between eclipses:

IV. $$l=a-b\cos\theta-ac\cos^2\theta+bc\cos^3\theta-a\frac{c^2}{2}\cos^4\theta$$

and

$$dl=da-\cos\theta\cdot db-a\cdot\cos^2\theta\cdot dc.$$

Intensities may be expressed in any unit. The average intensity between eclipses was taken as the unit. Twenty-two supernormals during primary eclipse, with the twenty already formed from the rest of the curve, gave forty-two equations for the determination of the 7 unknowns. The manner in which the normals are assembled to form the supernormals is indicated in the second

column of Table II, where the current number of the supernormal containing the normal is given. Normal No. 30, which can be represented by no set of unknowns, was finally omitted. The equations were divided through by the intensities to equalize the weights. The details of the observations and the least-squares solution will be found in a forthcoming contribution from the Princeton University Observatory. The solution resulted in the following set of values:

$$\begin{aligned} A &= 0.02285 \pm 0.00014 \\ B &= 0.01898 \pm 0.00021 \\ k &= 0.6244 \pm 0.0139 \\ \lambda_1 &= 0.1569 \pm 0.0015 \\ a &= 1.0175 \pm 0.0037 \\ b &= 0.0081 \pm 0.0027 \\ c &= 0.0330 \pm 0.0053 \end{aligned}$$

Since 4 of the unknowns, dA, dB, $d\lambda_1$, and dk, do not appear in the fourteen equations outside of the eclipses, the probable errors of these quantities should not be increased, to the full extent, by the very large residuals of asymmetry which prevail in these fourteen equations. The probable errors of these four unknowns as functions of the remaining three were determined from the twenty-five equations in which they appear. In combination with the probable errors of da, db, and dc, found from all thirty-nine equations, the probable errors of dA, dB, $d\lambda_1$, and dk were derived. The probable error of one normal point, usually comprising five observations, is ± 0.023. This is not excessively large, considering the residuals of asymmetry between eclipses. The residuals of the normals, O.−C., are given in the sixth column of Table II. This solution is also the basis of the curve drawn through the plotted observations.

The observations show strong and well-determined asymmetries and irregularities too large to be attributed to errors of observation. There is a well-defined hollow just after primary and another just after secondary. The most conspicuous asymmetry is a greater brightness from middle of primary eclipse through to secondary than on the other side. The secondary minimum gives no evidence of an eccentric orbit and the consequent possible explanation as a periastron effect. The term $0.017 \sin \theta$ takes care of the greater

part of this asymmetry between eclipses. The existence of this sine term would probably mean that the advancing side of the bright star is brighter than the following.[1] Stebbins[2] finds a similar asymmetry in the light-curve of δ Orionis, only in this case both components, which do not differ greatly in brightness, are apparently brighter on the advancing sides. An asymmetry in the same sense was found persisting throughout the entire light-curve of RT Persei.[3] To gain an idea of the effect of such an explanation on the residuals of the curve of RV Ophiuchi, I have taken the coefficient as determined between eclipses and assumed that at conjunction the visible disk of the bright star is divided into two equal areas of unequal brightness. The difference in total light of the two halves is 0.015 of the total light of the two stars. The new residuals, O.–C.′, Table II, show what improvement has been made by this partial explanation of the asymmetry. The sum of the weighted squares of residuals has been reduced from 1.143222 to 0.968367, a reduction of 15 per cent. Of the remaining sum 42 per cent is found among the 38 normals during primary eclipse, where, with few changes in sign, there has been a reduction of 21 per cent. The reduction outside of primary minimum is 10 per cent. Evidently the primary minimum requires a larger correction of the same character, while the stretches between eclipses require some explanation of the hollows in the curve. Some improvement during primary minimum could be effected by supposing that the brightness of the disk diminishes steadily from one limb to the other. On the whole, there seems to be some evidence that the advancing side of the brighter component of some eclipsing variables is brighter than the following side.

The elements of the system RV Ophiuchi follow:

Maximum radius of larger star	a_f	0.2069	±0.0010
Minimum radius of larger star	b_f	0.2001	
Maximum radius of smaller star	a_b	0.1291	±0.0006
Minimum radius of smaller star	b_b	0.1249	

[1] Announced at April 1915 meeting of American Philosophical Society; see *Proceedings*, **54**, 54, 1915.

[2] *Astrophysical Journal*, **42**, 142, 1915.

[3] *Contributions of Princeton Observatory*, No. 1.

Ratio of the radii of the stars	k	0.624 ±0.014
Ratio of the axes of the spheroidal stars	$1+\frac{1}{2}\varepsilon$	1.033 ±0.005
Least apparent distance of centers	$\cos i$	0.0709 ±0.007
Inclination of orbit plane	i	85°56′ ±24′
Eccentricity of orbit	e	0
Maximum fraction of light of smaller star obscured during eclipse	a_0	1
Interradiation difference of light of sides of larger star	$2b_1$	0.0176±0.0027
Interradiation difference of light of sides of smaller star	$2b_2$	0.0014
Light of brighter side of smaller star	L_b	0.825
Light of brighter side of larger star	L_f	0.175
Ratio of surface brightness		
Of the bright sides of the two stars	$\frac{J_b}{J_f}$	12.2
Of the sides of the fainter star		1.11
Difference of light of advancing and following sides of brighter star		0.015
Density of the brighter star	ρ_b	0.24
Density of the fainter star	ρ_f	0.06
(Assuming equal masses)		

SUMMARY

1. New elements for the occurrence of eclipses, containing a sine term of small amplitude, have been derived:

J.D. 2419808.81485+3^d.6871267 E−0^d.0014 sin 0°.22 E=1913 Feb. 9^{d}19^{h}33^m.4 G.H.M.T.+3^{d}16^{h}29^{m}27^s.75 E−2^m sin 0°.22 E.

2. During the total eclipse at primary minimum the star is 2^M.03 fainter than at maximum brightness, while the loss during the annular secondary eclipse amounts to 0^M.11.

3. The observed light-curve shows that RV Ophiuchi, a star of spectral type A, is an eclipsing system. One star emits nearly 5 times as much light as the other but its surface brightness is 12 times as great, the radius of the brighter star being but two-thirds that of the fainter star. The distance between centers is 5 times the radius of the fainter star.

4. The sides of the stars facing each other are found to be brighter than the opposite sides, the ratio being about the same as that found in the systems RT Persei and Z Draconis.

5. The stellar disks are found to be much more elliptical than was to be expected in a system where the radius of the stars is so small a fraction of the radius of the orbit.

6. Darkening toward the limb, if it exists in this case, is disguised by other phenomena. All the results contained in this paper are based on the assumption of uniform luminosity.

7. The light-curve is distinctly asymmetrical, the asymmetry persisting throughout the entire curve. This seems in part to be due to a difference in brightness of the advancing and following sides of the brighter star.

8. The theoretical curve which best represents the observations according to our present knowledge of the causes of light-variation in eclipsing systems still leaves residuals of a locally systematic character.

PRINCETON UNIVERSITY OBSERVATORY
December 4, 1915

A VARIATION IN THE SOLAR ROTATION

By H. H. PLASKETT

INTRODUCTION

For a number of years the spectroscopic determination of the solar rotation was in a rather unsatisfactory condition. In order to put matters on a better basis a co-operative investigation was inaugurated at the 1910 meeting of the International Solar Union. The spectrum was divided into a number of regions, one of which was assigned to each observer. While results from all the observers have not yet been published, still those results that have appeared show very large systematic differences. Accordingly, at the Bonn meeting of the Solar Union the following resolution was adopted:

> It is highly desirable to trace to their source the systematic differences that are found in the values of the solar rotation by different observers, and this investigation should take precedence of a continuation of the program adopted in 1910. For this purpose study should be directed to determining the velocity at the solar equator by as many different methods as possible, and it is recommended that the ten lines chosen in 1910 in the region λ 4220 to λ 4280 be used for this purpose. Investigation should also be made into the personal differences that are found in the measures of the same plate by different observers.

This resolution indicates very clearly the present status of the subject. That these "systematic differences" may be ascribed to instrumental or personal errors, or even to a variation in the rate of rotation of the sun, makes the matter most puzzling.

In order to test for the source of these differences, the writer, under favorable instrumental conditions, made, during the summer of 1915, a new series of plates, which were measured in duplicate. The methods used and the results obtained will now be discussed.

SECTION 1. METHODS

Since previous work of the writer had pointed in that direction, it was decided that a test should be made for a variation in the solar rotation. In order to afford the best opportunity for detecting

such a variation, certain precautions, to be described in full later, were considered in order that instrumental and personal errors might be reduced to a minimum. It was early recognized that this variation might occur in three ways: (1) rapid changes in the rotation rate occurring within a day; (2) slower changes in rate extending over several weeks; (3) a secular change in the rate. In order to cover these three possibilities the following program was drawn up: At least three equatorial plates were to be taken on each fine day for one or two months, and on at least four or five days a large number of plates were to be taken at frequent intervals for the greater part of the day as a test for rapid changes within a day. The means for all the plates from each day should give a good value of the velocity for that day, and these means should make possible a test of slow changes. Finally, a mean of all the plates should give a value of the equatorial velocity comparable with that obtained by the writer in 1913, thus permitting a test for secular change.

INSTRUMENTAL CONDITIONS

The instrumental equipment was practically the same as that hitherto used at Ottawa. A concave mirror of focal length 24.5 m is fed by coelostat and a secondary flat. It forms an image of the sun 23 cm in diameter on the face of a Littrow spectrograph, the beam having first passed through a louvred tube to the basement of the observatory. The optical parts of the spectrograph are a 15 cm Brashear lens of 7 m focus and a plane grating ruled by Anderson, used in the third order. The light from the two limbs of the sun is brought by reflecting prisms to another pair of two-toothed reflecting prisms directly over the slit. Hence on the photographic plate four spectra are obtained in the order east, west, east, west.

The errors to which spectroscopic observations of the solar rotation are subject fall into two main classes: those due to causes operating outside the spectroscope and those due to instrumental conditions within the spectroscope. In the first class are those errors produced by haze and by changes of focus and astigmatism of the mirrors. In the second class the chief source of error prob-

ably lies in unequal illumination of the grating by the two limbs. Flexure and change of temperature, which are so important in determinations of stellar radial velocity, play no part in solar rotation, since both limbs are affected equally. In this investigation special precautions were taken to insure that these sources of error were nullified. A brief description of the methods used will be of interest.

Of the sources of error external to the spectroscope, that due to haziness was carefully avoided. Haze, by diffuse reflection, produces blended spectra which give rotation values that are too small. Observations were never made except on days that were free from haze.

The errors due to astigmatism and change of focus were more serious. Within a few days after the mirrors had been freshly silvered it was found that they became unevenly heated. As a consequence, the image became astigmatic while the focus would lengthen abnormally. In order to reduce these difficulties an electric fan was installed which played upon the coelostat mirror. This was found to give very satisfactory results, not only as regards astigmatism, but also in keeping the focus constant.

When the fan was installed and it was seen that the focus remained constant it was decided to determine the value of R, the diameter of the sun in millimeters, by two independent methods: (1) to measure R directly at the proper focal point; (2) to determine the focal length of the mirror for each exposure, and, from the semi-diameter of the sun in seconds for that day, to compute R in millimeters. The two values of R thus obtained were found to be in close agreement over the entire series of observations, the mean numerical residual being of the order of half a millimeter, or considerably less than one-fourth of 1 per cent.

As was mentioned before, the only important source of error internal to the spectroscope is that due to uneven illumination of the grating from the two limbs of the sun. Of course, if the emulsion of the photographic plate could be kept exactly at the focus of the camera lens the spectra from the two limbs would have no relative displacement, but it seems almost impossible to do this. Owing probably to uneven heating of the cells holding the prisms

it is quite possible for the illumination to change during the course of an exposure and still no change be noticeable when the illumination is tested. It is this source of error, more, probably, than any other, which tends to vitiate the results in spectroscopic determinations of the solar rotation.

The writer had early decided that the best plan was to use a region of the spectrum where there were sufficient telluric lines to act as a check. It is evident that if the spectra from the two limbs are displaced relatively to each other the telluric lines will show a like shift. Thus, by measuring both the solar and telluric lines and applying the proper signs the algebraic sum of their means should give the true radial velocity. In order to carry out this plan it was decided to take the plates in the region of λ 5900, where there are a number of very well defined water-vapor lines.

Trouble was early experienced in securing plates sensitive to the region. Several well-known brands of commercial panchromatic plates were tried, and, though fairly satisfactory results were secured as regards grain and contrast, yet the exposure time was excessive (something over twenty minutes). Accordingly, the writer made some experiments with stained plates. The usual pinachrome solution[1] was tried on Seed Contrast Process plates. The result was in every way more satisfactory, though the exposure was still too long (fifteen minutes). Some Cramer Iso Process plates were then stained. Most satisfactory results as regards both grain and contrast were secured, while at the same time the exposure was cut down to two minutes. Accordingly, with the exception of the first three or four, which are stained Seed Contrast Process, the plates used in this investigation were Cramer Iso Process stained with pinachrome.

Having in this manner secured a check on the chief instrumental sources of error, the program previously outlined was carried out. On every fine day, the sky being free from haze, from three to four plates were taken at the equator. The plates were taken early in the morning, between five and eight o'clock, in order that the telluric lines might be at their greatest strength. By a plate is

[1] *British Journal and Photographic Almanac*, p. 595, 1915.

meant here, and in subsequent writing, one exposure showing four spectra, two from each limb. Thus a number of exposures may be secured on one plate, but for convenience in discussion each one of these exposures will be termed a plate. In this manner 111 plates were secured on 25 days, an average of a little over four plates per day.

MEASUREMENT AND REDUCTION OF PLATES

Owing to the large number of plates to be measured, and to the fact that they were to be measured in duplicate for a reason to be explained later, it was decided to measure the displacements of only five solar and three telluric lines. From earlier work it was evident that a good value of the rotation would be given by five lines. In Table I will be found the wave-lengths, the elements to which the

TABLE I

LINES IN THE REGION OF λ 5900

No.	λ	Element	Int.	1 mm in Angstrom Units	Vel. Const.
1	5857.674	Ca	8	0.69189	17.709
2	5862.582	Fe	6	.69146	17.683
3	5866.675	Ti	3	.69112	17.662
4	5886.193	A(wv)	5	.68944	17.561
5	5916.475	Fe	3	.68685	17.406
6	5919.860	A(wv)	7	.68655	17.388
7	5932.306	A(wv)	5	.68547	17.324
8	5934.881	Fe	5	0.68526	17.311

lines are due, and the intensities of these eight lines taken from Rowland's tables. The value of one millimeter in angstrom units was obtained from the mean of five measures of the spectrum of the sun's center. The values were smoothed out by a curve. In the last column of the table will be found the velocity constant. This when multiplied by the displacement in millimeters gives the radial velocity due to the rotation of the sun.

The relative displacements of the lines in the spectra of the two limbs were measured. One setting was made in each strip of spectrum, so that two settings were made on the line from each limb. The plate was then reversed and this procedure repeated. Thus eight settings in all were made on each line on each plate.

Telluric displacements were counted positive when they were in the opposite direction to the solar displacements. Hence the algebraic sum of the mean solar and telluric velocities should give the velocity free from instrumental error. The error involved in multiplying the constant telluric displacement, if it exists, by a "velocity constant" varying with the wave-length is of course negligible.

From the writer's previous experience with the possibility of variation[1] in the personal equation, it was decided to measure the plates in duplicate. No plates were measured until all the plates were taken. They were then mixed at random and divided into two bundles. One bundle was measured on the Repsold machine with an eyepiece micrometer and the other on a Toepfer machine with a 300 mm screw. As a plate was measured on one machine it was placed at the bottom of the bundle for the other machine. As a result the maximum time elapsed between two measures of the same plate. As no reduction was made until the measuring was completed, it was impossible for any prepossession to enter the result. Furthermore, since the telluric lines were measured at the same time as the solar, it seems not unlikely that if, for example, the solar displacements were too large, the telluric lines would be measured negatively, and hence an automatic correction would be carried along. The final result thus ought to be free from personal equation as well as instrumental error.

The preliminary reduction was carried out as follows: The mean solar velocity was secured from the five solar lines, and likewise the mean telluric velocity with its proper sign was obtained. Their algebraic sum gave the preliminary v for each plate. As each plate was measured twice, it was the mean of the two measures that was used in the final reduction to the true sidereal rate.

SECTION 2. RESULTS

The unreduced values of the velocity for the 111 plates are given in Table II. In the first column will be found the date G.M.T. at which each plate was taken. In the second column is given the

[1] "The Solar Rotation in 1913," *Journal R.A.S. of Canada*, 7, 307, 1914; "The Psychology of Differential Measurements," *ibid.*, 9, May 1916.

difference between the two measures of the plate, viz., that made on the Repsold machine and that made on the Toepfer. Finally, the means of the two measures are given in the third column.

TABLE II

MEASURES

Date G.M.T. 1915	I–II	v	Date G.M.T. 1915	I–II	v	Date G.M.T. 1915	I–II	v
	km	km		km	km		km	km
171d994	−0.015	1.745	192d991	−0.049	1.813	210d023	+0.001	1.820
172.008	+ .116	1.819	.993	+ .023	1.817	.061	− .148	1.760
.022	− .051	1.699	.999	+ .130	1.912	.063	+ .012	1.859
.036	− .052	1.723	193.002	+ .073	1.820	.065	+ .078	1.851
174.128	− .097	1.710	194.994	+ .104	1.879	211.008	− .040	1.718
.146	+ .017	1.725	.999	− .061	1.838	.010	+ .097	1.733
.163	− .062	1.736	195.004	− .029	1.819	.012	− .016	1.715
175.066	− .008	1.658	.008	+ .030	1.846	222.073	+ .062	1.794
.083	+ .051	1.700	200.975	− .126	1.839	.075	+ .044	1.799
176.028	− .005	1.787	.988	+ .027	1.869	.078	− .043	1.800
.060	+ .060	1.702	.993	+ .115	1.838	.080	+ .196	1.755
.079	− .017	1.735	201.045	− .125	1.892	.105	− .056	1.800
177.057	+ .091	1.645	.987	+ .027	1.843	.109	+ .140	1.742
.076	+ .087	1.754	.989	+ .031	1.778	.111	+ .123	1.797
179.044	+ .075	1.671	.997	+ .082	1.799	.132	− .074	1.789
.064	− .007	1.738	202.000	− .039	1.709	.134	+ .084	1.885
.083	+ .039	1.714	204.043	− .103	1.871	.137	+ .022	1.757
182.036	+ .063	1.709	.046	+ .033	1.874	223.156	+ .097	1.805
.042	+ .024	1.735	.049	+ .033	1.774	.157	+ .091	1.852
.051	+ .087	1.734	.051	+ .038	1.842	.160	+ .021	1.846
.058	+ .002	1.660	205.048	− .008	1.749	226.003	+ .005	1.843
184.960	+ .063	1.760	.050	+ .030	1.754	.006	+ .103	1.822
.967	+ .009	1.705	.053	+ .070	1.805	228.070	+ .079	1.824
.974	+ .039	1.765	.055	− .008	1.821	.072	+ .061	1.841
.985	+ .050	1.704	207.966	− .030	1.765	.074	− .126	1.778
190.045	− .024	1.729	.968	− .046	1.884	.106	+ .064	1.824
.049	+ .141	1.775	.971	+ .062	1.880	.108	+ .020	1.789
.052	+ .003	1.814	208.010	+ .009	1.790	.110	− .020	1.833
.056	+ .100	1.699	.013	− .019	1.828	.133	− .067	1.851
190.965	+ .044	1.850	.979	+ .024	1.800	.137	− .032	1.738
.968	− .029	1.851	.983	+ .035	1.788	.139	+ .075	1.782
.973	− .015	1.796	.988	− .013	1.800	.168	+ .011	1.823
.976	+ .036	1.873	209.977	+ .098	1.721	.170	+ .143	1.836
192.001	− .017	1.822	.979	+ .012	1.731	.172	+ .010	1.786
.005	+ .010	1.834	.982	+ .051	1.816	.196	+ .113	1.892
.009	− .102	1.831	210.019	− .146	1.801	.198	− .001	1.773
.013	−0.010	1.782	.021	+0.027	1.861	.201	−0.045	1.762

ERRORS OF MEASUREMENT

It will be of interest to compare the two series of measures. Since the measures are made of the same plates, it is evident that they should agree within the limits of accidental error. Consider-

ing the residuals as a whole, it is seen that they are in fair agreement. The algebraic mean of I–II is +0.020 km. That is, there is a constant difference between the two series which must be ascribed to a difference in personal equation of the writer with the two machines. Eliminating this constant difference and taking the numerical mean of the residuals thus left, the mean becomes 0.021 km. This number, 0.021 km, may be taken as a measure of the accidental differences between the two series, and it shows that the two series are in satisfactorily close agreement.

However, it will be noticed that there are several very large residuals (greater than a tenth of a kilometer) in the second column. Allowing for the constant difference, there are 14 residuals out of 111, or 13 per cent of the residuals are greater than a tenth of a kilometer. At first sight it is difficult to see how such large differences could have arisen in measuring the same plate. The writer is of the opinion, however, that it is due very largely to the measures of the small telluric displacements. An examination of the measures shows that in 11 cases out of the 14 it was the telluric velocity that had changed its value by more than a tenth of a kilometer, while the solar velocity had remained the same in the two series. A complete discussion of the whole matter will be found in a recent article[1] by the writer, in which he shows that the habit of measurement is subject to very rapid changes in measuring these smaller displacements.

So much for a comparison of the two series of measures. When the difficulties in the measurement of the telluric displacements are taken into consideration it is readily seen that the two series are in satisfactorily close agreement. Accordingly, in obtaining the final results the mean of the two measures for each plate was used. This mean is given in the third column of Table II.

Again, it is readily seen that while the plates for each day give a fairly homogeneous mean, yet there are some large discrepancies. It is interesting to note that these large discrepancies are accompanied by correspondingly large residuals, showing that they are due to errors of measurement. However, on the whole the results for each day are fairly homogeneous. Taking the mean for each

[1] "The Psychology of Differential Measurements," *op. cit.*

day and then the probable error of that mean, it is found that the mean of the probable errors is +0.012, a satisfactorily small quantity.

VARIATION IN THE SOLAR ROTATION

After this preliminary discussion it is now possible to apply the test for a variation in the rate of rotation. As previously mentioned, this variation may take three forms: (1) rapid changes within a day; (2) slower changes, extending over several weeks; and (3) a secular change since 1913.

Variation within a day.—Unfortunately, owing to the unexpectedly large errors of measurement, it is impossible to give a definite answer to this question. However, large as these errors are, it may be safely said that were there a range of values within the day as great as a tenth of a kilometer it would be readily evident.

Variation over several weeks.—In order the better to test for this variation Table III was formed. Columns one, three, and four

TABLE III

SUMMARY OF RESULTS

Date G.M.T. 1915	No. Plates	I–II	v	V	P.E.	ξ
		km	km	km	km	°
$172^d.02$	4	−0.001	1.746	1.911	±0.018	13.57
174.15	3	− .047	1.724	1.885	.005	13.38
175.07	2	+ .022	1.679	1.846	.015	13.11
176.06	3	+ .013	1.741	1.914	.016	13.59
177.07	2	+ .089	1.700	1.866	.038	13.25
179.06	3	+ .036	1.708	1.875	.014	13.31
182.05	4	+ .043	1.709	1.871	.012	13.28
184.97	4	+ .041	1.734	1.893	.012	13.44
190.05	4	+ .055	1.754	1.919	.017	13.62
190.97	4	+ .009	1.843	2.009	.011	14.26
192.01	4	− .030	1.817	1.975	.008	14.02
193.00	4	+ .044	1.840	2.000	.016	14.20
195.00	4	+ .011	1.845	2.007	.008	14.25
201.00	4	− .027	1.860	2.026	.008	14.38
201.99	4	+ .025	1.782	1.943	.019	13.80
204.05	4	+ .000	1.840	2.003	.016	14.22
205.05	4	+ .021	1.782	1.951	.013	13.85
207.99	5	− .005	1.829	1.996	.016	14.17
208.98	3	+ .022	1.796	1.954	.003	13.87
210.02	9	− .002	1.802	1.964	.013	13.94
211.01	3	+ .014	1.722	1.885	.004	13.38
222.10	10	+ .050	1.792	1.966	.008	13.96
223.16	3	+ .070	1.834	2.007	.010	14.25
226.00	2	+ .053	1.832	2.006	.007	14.24
228.14	15	+0.019	1.809	1.977	0.007	14.04

are the same as in Table II except that they give the mean for each day. The number of plates used in forming this mean are given in the second column. In the fifth column is given the final mean reduced velocity for each day. The probable error of this mean is given in the next column. For completeness the probable error is given for each day, though, when only two plates are used in forming the mean, it has not much significance. In the last column is given the angular velocity corresponding to V.

Using the dates in this table as abcissas and the final reduced values, V, as ordinates, Fig. 1 has been constructed. The value of the velocity for each day has been represented by a circle. Owing to rainy weather there are, unfortunately, no observations

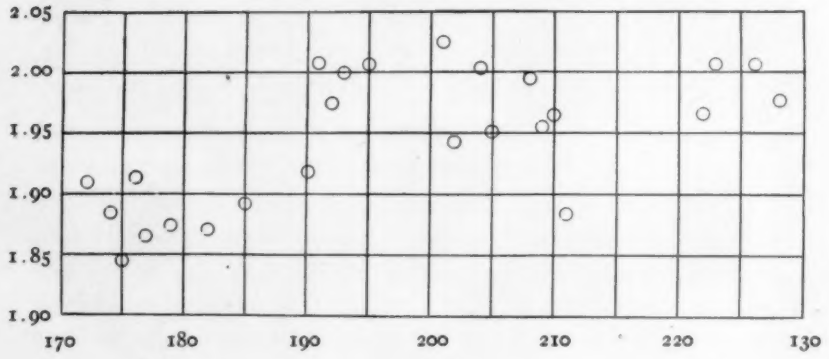


FIG. 1

in the interval 211–222 days. Notwithstanding this, the figure shows quite clearly a cyclic variation with a range of 0.15 km. From a velocity of 1.91 km on the 172d day the rate drops to 1.87 km on the 177th day. It then rises slowly to a maximum of 2.00 km, about which it hovers for nearly two weeks. It then drops quickly to a minimum about the 211th day. At this point rainy weather intervened, and no more observations were secured until the velocity was once more a maximum.

Owing to the short interval covered by the observations it is impossible to draw any conclusions as to the periodicity of this phenomenon. The most that can be said is that during the summer of 1915 the rate varies cyclically in about a month. In order

to show this cyclic variation better the writer has grouped the observations in the interval 172–211 days into 14 normal places. The results are shown graphically in Fig. 2.

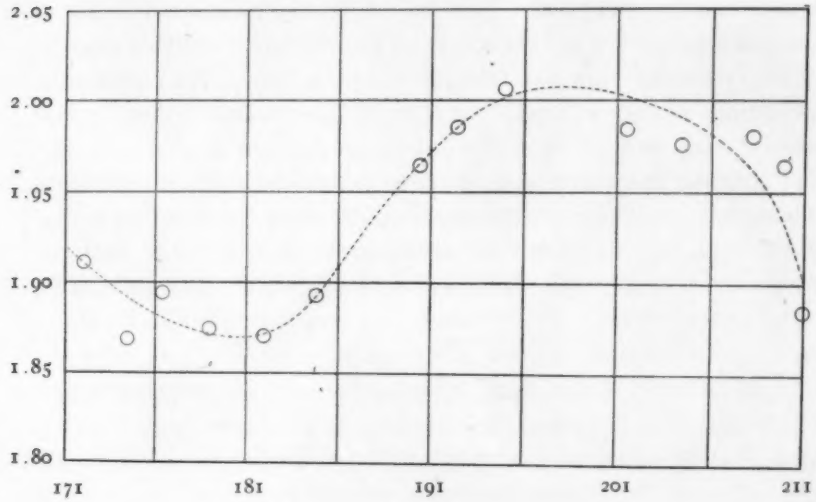


FIG. 2

Secular variation.—The mean equatorial value of the solar rotation during the time the sun was under observation was obtained by taking a simple mean of the values of V in Table III. This gives

$$V = 1.949 \text{ km per sec.}$$
$$\xi = 13^{\circ}.83 \text{ per day}$$

Comparing these with the values obtained by the writer in 1913, viz.:

$$V = 1.991 \text{ km per sec.}$$
$$\xi = 14^{\circ}.14 \text{ per day}$$

it is evident that there has been a real change in the interim.

SECTION 3. DISCUSSION OF RESULTS

Before proceeding with this discussion it is of interest, in view of what follows, to know the effective level at which the observations were made. The effective level of the lines, as taken from the results of Mitchell's observation of the flash spectrum, is about 400 km.

CYCLIC VARIATION

In the previous section it was shown that the sun, during the summer of 1915, underwent a cyclic variation in its rotation rate with a range of 0.15 km. This variation was completed in about a month; nothing could be said as to the periodicity of this change.

At first sight it seems strange that a variation of this magnitude, involving a change of nearly 2.4 days in the rotation period, should have so long escaped detection. Second thought shows, however, that it is not so strange as it appears. Until recently observations of sun-spots were the only means of determining the rotation period of the sun. Observations extending over two or three sun-spot cycles have been made by Carrington, Spoerer, and Maunder. In a recent discussion of the Greenwich sun-spot records, Maunder says: "The rotation periods given by different spots in the same zone of latitude differ more widely than do the mean rotation periods for different zones of latitude. It has been forgotten that whatever the cause which produces the variation of rotation rate with latitude, the causes producing differences of rate within any given latitude are more effective still."[1] With such variable standards it would be almost impossible to detect a cyclic change of this magnitude, particularly if the interval required to complete a cycle coincided closely with the rotation period of the sun. The same conclusions may be applied with even more point to determinations of the rotation with either faculae or flocculi.

These objections have no force, however, in connection with spectroscopic determinations of the solar rotation period, and it seems strange that this cyclic variation has not been detected before. Strangely enough, previous observations have shown nearly the same range as that obtained by the writer, but unfortunately the observations have not been properly grouped to show a variation. As a rule, a few plates have been taken spread over long intervals, with rarely more than one plate exposed on a day. As a consequence accidental errors tend to cloak any real variation that might exist. Fortunately the plates taken at Ottawa in 1912[2] form an exception, since they are so grouped as to make a test possible. In

[1] *Monthly Notices*, **65**, 823, 1905.

[2] *Astrophysical Journal*, **42**, 373, 1915.

Table IV will be found the date G.M.T., the number of plates used in forming the mean, the mean reduced velocity, and its probable

TABLE IV

λ 5600, 1912, J. S. PLASKETT

Date G.M.T. 1912	No. Plates	*V*	P.E.
		km	
158d.20.....	1	2.055	.....
159.20.....	3	2.002	+0.009
161.15.....	3	1:992	.014
166.27.....	4	2.039	.003
167.18.....	1	1.969	.....
170.21.....	3	1.984	.008
179.29.....	7	2.011	.005
180.12.....	3	2.054	.002

error. These results are shown graphically in Fig. 3. Evidence of a variation of the same type as that in Fig. 2 is shown here. The λ 4250 results also show a variation of this character which

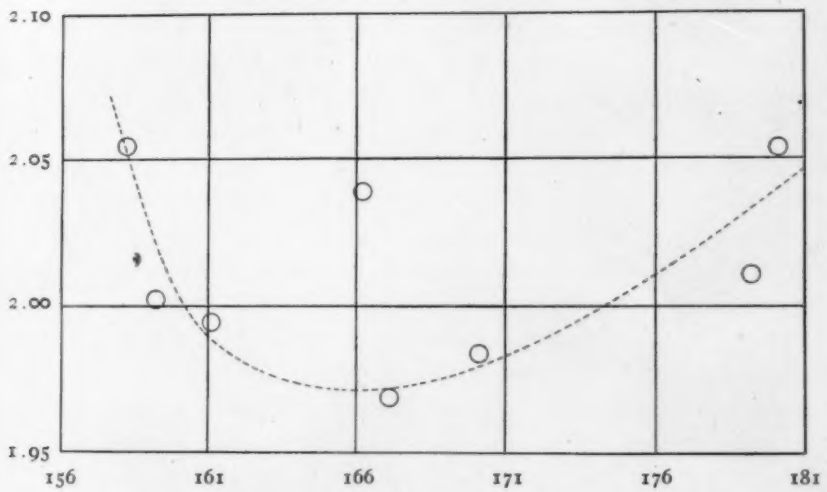


FIG. 3

can readily be seen by consulting Table IV of the original paper. In this case the values fall to a minimum at the end of the series of plates.

It is unfortunate that previous work does not throw more light upon this cyclic variation. Observations will have to be extended over several years before data can be gathered sufficient to explain this phenomenon. It is of interest, however, and also important to realize that a physical explanation can be found. Variation in the rotation of the sun, assuming its moment of momentum constant, can be caused by oscillations about its mean form. Moulton[1] has treated these oscillations fully and has shown, in the case of the sun, that the shrinkage of even a second of arc in the diameter will cause a change in diurnal rate of only 0°.02. One-tenth of this shrinkage would cause the sun to change in brightness by a whole magnitude, and hence it is evident that the cyclic variation is not caused by oscillations. Emden's solar theory[2] calls for a "velocity of rotation variable in both time and space." The homogeneous layers, each with its own angular momentum and potential temperature, assumed by him to account for the heat exchange between center and surface, explain at the same time the equatorial acceleration. It is evident, if this theory be true, that at any given level the velocity should rise slowly to a maximum and fall abruptly to a minimum. The interval of time covered by this change will depend upon the thickness of the layer. For, consider a warm layer at the given level. As it cools by external radiation it contracts, and, as it contracts, according to Emden, it accelerates. This acceleration will continue until this layer has sunk below the given level (400 km), and a new warm layer with a low angular velocity has appeared. This is the type of variation shown in Fig. 2: a slow rise to maximum and a sudden fall to minimum. This simple process will of course be greatly complicated by exchange of heat between adjacent layers. Sufficient has been said, however, to show, provided Emden's theory be correct, that the phenomenon admits of a physical explanation, and also to indicate that its final elucidation is largely a question of sufficient data.

SECULAR VARIATION

A change in velocity of this nature has long been suspected. Thus Halm thought he had evidence of it in his observations at

[1] *Astrophysical Journal*, **29**, 257, 1909. [2] *Ibid.*, **15**, 38, 1902.

Edinburgh. However, as evidence accumulated it began very generally to be felt by astronomers that the evidence in favor of a secular change was inconclusive.

This view is of course greatly strengthened by the fact that observations of the solar rotation from sun-spots show no evidence of secular variation. To quote Maunder again: "There is no clear evidence of anything like a systematic change in the rotation period during the progress of any particular cycle, and very little evidence, if any, of a change from one cycle to another."[1] Evidence such as this may be taken as conclusive as regards a secular change in rate at the sun-spot level. For, though with regard to cyclic changes, sun-spots, because of their proper motions, can furnish no definite evidence, yet the mean over a year should give a good value of the rate for that year.

As a consequence it must be concluded that this secular change is confined to the reversing layer. It is of interest to note that the low value obtained by the writer in this paper is confirmed in a recently published abstract[2] from Mount Wilson. The velocity obtained at that institution during the years 1914 and 1915 is

$$V = 1.949 \text{ km per sec.}$$

Though previous observations are not strictly comparable on account of the possibility of instrumental and personal differences, yet it appears that since 1906 the rotation value has been slowly falling from 2.050 km to 1.950 km.

Here again elucidation of this secular change must await the gathering of more material, as even less is known of the secular change than of the cyclic variation. Observations ought to be continued by all institutions that have been engaged in this work, and measurements, wherever possible, ought to be made by the same observer. In this manner a large quantity of data free from systematic differences could readily be gathered.

CONCLUSIONS

1. A refinement of instrumental conditions led to results practically free from instrumental error.

[1] *Op. cit.*

[2] *Popular Astronomy*, **23**, 641, 1915.

·2. Measures of each plate were made in duplicate. On account of the difficulty of measurement of the small telluric displacements there were a few large accidental errors.

3. The results were inconclusive as regards variation within a day, if the range be less than one-tenth of a kilometer. There was no evidence of a variation with a range greater than this.

4. There was good evidence of a cyclic variation extending over a month with a range of 0.15 km. Too short an interval was covered by the observations to make possible any statement as to periodicity.

5. The mean of all the measures showed that the rotation in 1915 was 0.04 km lower than it was in 1913.

6. These results were in general confirmed by a discussion of previous spectroscopic determinations of the solar rotation.

In conclusion the writer would like to acknowledge the kindly interest shown and the encouragement afforded by the director, Dr. W. F. King, during the progress of this investigation.

DOMINION OBSERVATORY, OTTAWA
December 1915

Minor Contributions and Notes

ON THE POLE-EFFECT IN A CALCIUM ARC

In the course of an investigation of the pressure-shift of the lines in the arc spectrum of calcium, which is in progress, it seemed desirable to make some observations on the pole-effect of the triplets of the first and second subordinate series.

A horizontal arc, 4 mm long and carrying 4 amperes on a 110-volt circuit was used, with electrodes of metallic calcium, varying in diameter from 7 to 10 mm.

The image of the arc was projected with a magnification of about 3 upon the slit of a 21-ft. concave grating spectrograph, and the spectrum in the first order from λ3000 to λ4200 was photographed on a single plate. This region includes, according to Kayser's notation, IIN4, IN5, IIN5, and IN6, to which our measures have been confined. The scale of the plates is 2.64 A per mm.

Table I presents the data for the metallic arc between calcium poles:

TABLE I

POLE-EFFECT AT 1 ATMOSPHERE. CALCIUM ELECTRODES

Positive minus Center

	IIN4	IN5	IIN5	IN6
Mean.....	+0.008 mm (28)	−0.009 mm (21)	+0.014 mm (7)	−0.011 mm (6)

Negative minus Center

	IIN4	IN5	IIN5	IN6
Mean.....	−0.002 mm (26)	+0.004 mm (27)	−0.004 mm (7)	+0.009 mm (7)

The numbers in parentheses indicate the number of measures averaged.

The change of sign with the series is very striking, and apparently a continuous change in wave-length is indicated from one pole to the other. From a qualitative standpoint, the conclusions, we think, are quite definite, but on account of the unsteady

conditions in the arc between calcium electrodes no great weight can be attached to the numerical values.

It is noteworthy that in the case of the calcium arc the shift in going from the center to the positive pole is opposite to that

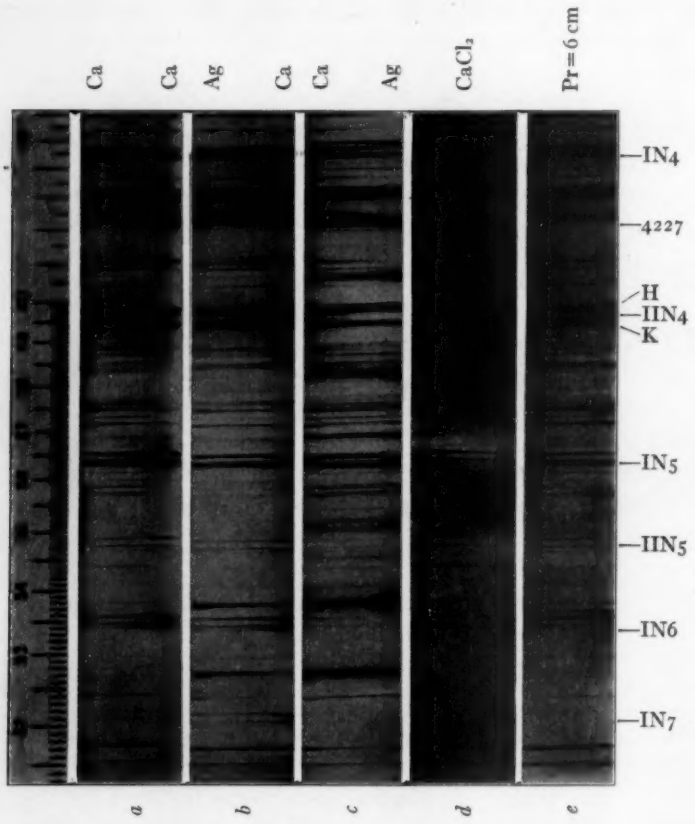


FIG. 1

obtained in going from the center to the negative pole. As has been observed by Goos[1] and by St. John and Babcock,[2] this is not the case with the lines of the *d* and *e* groups in the spectrum of iron, where the shift in going from the center of the arc to the positive pole is in the same direction, though of a smaller amount, as that found in going from the center of the arc to the negative pole. St. John and Babcock state that a longer exposure time is required

[1] *Astrophysical Journal*, 38, 141, 1913. [2] *Ibid.*, 42, 231, 1915.

at the center of the arc than at either pole in order to secure lines of the same strength. We had noted that a longer exposure was required at the negative pole than at the center of the arc, and a longer exposure at the center than at the positive pole, in the case of the subordinate series lines of calcium. We therefore projected the image of our calcium arc on the slit of a quartz spectrograph. When both poles were calcium, we found that the lines of the first and second subordinate series were strongest at the positive pole and diminished continuously in intensity from the positive pole to the negative, as shown in Fig. 1*a*.

Since the wave-length appeared to depend upon the intensity of the part of the line from which the light is taken, it seemed desirable to vary, if possible, the intensity-gradients across the arc and examine the corresponding pole-effects. With the positive pole calcium, and the negative pole silver, Fig. 1*b*, the intensity-gradient for the lines under investigation was the same as when both poles were calcium; but with clean Ag+ and Ca−, the gradient was reversed (Fig. 1*c*), and the pole-effect was also reversed, as shown in Table II.

TABLE II

POLE-EFFECT AND REVERSED INTENSITY-GRADIENT

Silver+ and Calcium−

Negative minus Center

IIN_4	IN_5	IIN_5	IN6
+0.008 mm	−0.011 mm	+0.017 mm	− 0.015 mm
+0.006	−0.012	.....	.....

When ordinary carbon rods were charged with calcium by immersing them in solutions of $CaCl_2$, the intensity-gradients were as shown in Fig. 1*d*, i.e., strongest at the negative pole and diminishing to the positive pole. This is opposite to the intensity-gradient when both poles are metallic calcium; and the pole-effect is also opposite, as may be seen in Table III.

St. John and Babcock found that the pole-effect disappeared *in vacuo*. We have also found that with calcium electrodes at a pressure of about 6 cm, the intensity-gradients are as shown in Fig. 1*e*. It is not easy to secure consistent measurements of the

pole-effect with electrodes of metallic calcium either *in vacuo* or at atmospheric pressure, since the arc does not burn steadily, but the general conclusion seems to be that the pole-effect in this case is zero or very small, as shown in Table IV.

TABLE III

REVERSAL OF POLE-EFFECT WITH INTENSITY-GRADIENT

Vapor-Density and Pole-Effect. Carbon Poles Charged with $CaCl_2$. Density-Ratio = 20 to 1

Negative minus Center

	IIN4	IN5	IN6
Denser vapor	+0.005 mm	−0.007 mm	−0.019 mm
	+0.004	−0.008	−0.021
	+0.003	−0.007	.....
	+0.004	−0.005	.....
Mean	+0.004	−0.007	−0.020
Rarer vapor	+0.004	−0.006	.....
	+0.003	−0.005	.....
Mean	+0.004	−0.006	.....

The employment of carbon electrodes charged with $CaCl_2$ in solutions of different concentrations presents an opportunity of demonstrating the effect of vapor-density on the pole-effect, and

TABLE IV

POLE-EFFECT *in vacuo*. CALCIUM ELECTRODES

Positive minus Center

IIN4	IN5	IIN5	IN6
0.000 mm	−0.002 mm	−0.001 mm	0.000 mm
+0.002	+0.005	+0.001	+0.001
+0.001	−0.002	.....	−0.002
−0.002	−0.004	.....	−0.006
+0.005	+0.001	.....	−0.003
+0.001	−0.002	.....	0.000
0.000	.....	.....	.....
Mean.. +0.0008	−0.0003	0.000	−0.0016

also upon the absolute wave-length of these lines. The measures of Table III taken between negative and center with arcs whose relative vapor-densities, as judged from exposure-times, were

about 20 to 1 would seem to indicate that the magnitude of the pole-effect is independent of the density of the radiating ions. The effect of vapor-density on absolute wave-length was investigated by comparing the same part of carbon arcs charged with different amounts of $CaCl_2$, and from Table V it would seem that the absolute wave-length is unchanged by vapor-densities which differ by as

TABLE V

WAVE-LENGTH AND VAPOR-DENSITY

Carbon Poles Charged with $CaCl_2$.

Density-Ratio = 10 to 1

High Density minus Low Density

IIN4	IN5
+0.001 mm	+0.002 mm
−0.001	+0.002
+0.001	−0.002
−0.001	−0.001
0.000	0.000
Mean.. 0.000	0.000

much as 10 to 1. A similar conclusion was reached by St. John and Babcock regarding wave-lengths in the furnace spectrum of iron. The series lines of calcium here considered are well suited to this investigation since they are extremely sensitive to pole-effect and pressure-shift, and in these phenomena the lines of the first and second subordinate series show displacements of opposite sign.

TABLE VI

POLE-EFFECT AND PRESSURE-SHIFT

	IIN4	IN5	IIN5	IN6
Positive minus negative....	+0.010 mm	−0.013 mm	+0.018 mm	−0.020 mm
Pressure-shift × 1.7.....	+0.010	−0.012	+0.017	−0.022

We find, by a comparison with a preliminary determination of the pressure-shifts for these lines (Table VI) that there probably exists a parallel relationship between the two phenomena, as St. John and Babcock also observed for the iron arc. The displacements in both cases are in the direction of the unsymmetrical widening with increased intensity and pressure.

The conclusion indicated by the measures cited is that the pole-effect for the lines of the subordinate series depends upon the intensity-gradient from pole to pole across the arc, i.e., when light taken from any part of a line is compared with that taken from a stronger part of the same line, there is a difference in wave-length similar to that caused by an increase in pressure: the wave-length at the stronger part of the line corresponds to the wave-length at the greater pressure.

The fact that we obtained the same value for the wave-length with a small or a much larger amount of $CaCl_2$ present, the ratio of exposure-times being about 1 to 10, whereas the exposure-ratios between center and positive pole were about 1 to 2 or 3, indicates that mere density of the radiating ions is probably not the underlying cause of the pole-effect.

It therefore occurs to us that possibly the amplitude of vibration of the electrons is the factor which disturbs the wave-length. On this hypothesis, lines which are shifted to the red by pressure are increased in wave-length and lines that are shifted to the violet are decreased in wave-length with increasing amplitude of vibration. The well-known facts that increase of current causes changes in wave-length similar to pole-effect changes, and that the pole-effect is greater at the negative than at the positive pole in the iron arc, in which the negative pole is brighter although the positive pole is hotter, are consistent with this hypothesis. There may also be significance in the fact that the arc increases in intrinsic brightness with increase of pressure, which suggests the possibility that pressure-shift may also be associated with the amplitude of vibration of the electrons, since pressure-shift, for symmetrical lines at least, is independent of vapor-density. It will be recalled also, in this connection, that the pressure-shift of Ti lines has been found[1] to be 40 or 50 per cent higher in the Ti spark than in the arc.

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January 1916

[1] *Astrophysical Journal*, **35**, 10, 1912.

CORRECTIONS TO THE MEASURED VELOCITY OF A SPECTROSCOPIC BINARY, DEPENDING UPON THE LENGTHS OF EXPOSURE OF THE SPECTROGRAMS

If a spectroscopic binary is faint and its period of oscillation is short, it may happen that the length of exposure necessary to secure a measurable spectrogram will amount to a considerable fraction of the period. This case has arisen a number of times and is likely to occur more frequently in the future. Thus the binary 12 Lacertae has recently been shown by Dr. Young of Ottawa to have a period of less than five hours, and some of the spectrograms from which this result was deduced were exposed for more than half the period. This star is also being observed at Allegheny by Mr. Daniel, who confirms the short period. Some of the Allegheny spectrograms were exposed over two hours.

It is obvious that velocities derived under such circumstances need a correction depending upon the length of exposure. Let u_1 be the phase-angle at the beginning of the exposure and u_2 that at the end. Then in a circular orbit the true velocity at the mid-time of exposure is

$$K \cos \frac{u_2+u_1}{2}. \qquad (1)$$

Here and in what follows we count the velocity from that of the center of mass of the system, and we count u, as usual, from the ascending node. The measured velocity from this same spectrogram, aside from accidental errors, will be the mean velocity during the interval u_1 to u_2; that is

$$\frac{K}{u_2-u_1}\int_{u_1}^{u_2} \cos u \, du;$$

or

$$K \cos \frac{u_2+u_1}{2} \cdot \sin \frac{u_2-u_1}{2} \cdot \frac{2}{u_2-u_1}. \qquad (2)$$

Comparing this with (1) we see that the measured velocity needs the correction

$$K \cos \frac{u_2+u_1}{2}\left(1-\frac{2}{u_2-u_1} \sin \frac{u_2-u_1}{2}\right). \qquad (3)$$

The numerical values for this coefficient of K are given in Table I. The vertical argument is the mean phase, and the horizontal argument is the length of exposure expressed in fractions of the period of oscillation, and also in degrees of arc.

TABLE I

VALUES OF $\cos \frac{u_2+u_1}{2}\left(1-\frac{2\sin\frac{u_2-u_1}{2}}{u_2-u_1}\right)$

$u_2-u_1=$	0° 0.00	18° 0.05	36° 0.10	54° 0.15	72° 0.20	90° 0.25	108° 0.30	126° 0.35	144° 0.40	162° 0.45	180° 0.50	$=u_2-u_1$
$\frac{u_2+u_1}{2}$												$\frac{u_2+u_1}{2}$
0°	0.00	0.00	+0.02	+0.04	+0.06	+0.10	+0.14	+0.19	+0.24	+0.30	+0.36	360°
10	.00	.00	+ .02	+ .04	+ .06	+ .10	+ .14	+ .19	+ .24	+ .30	+ .36	350
20	.00	.00	+ .02	+ .03	+ .06	+ .09	+ .13	+ .18	+ .23	+ .28	+ .34	340
30	.00	.00	+ .01	+ .03	+ .06	+ .09	+ .12	+ .16	+ .21	+ .26	+ .31	330
40	.00	.00	+ .01	+ .03	+ .05	+ .08	+ .11	+ .15	+ .19	+ .23	+ .28	320
50	.00	.00	+ .01	+ .02	+ .04	+ .06	+ .09	+ .12	+ .16	+ .19	+ .23	310
60	.00	.00	+ .01	+ .02	+ .03	+ .05	+ .07	+ .09	+ .12	+ .15	+ .18	300
70	.00	.00	+ .01	+ .01	+ .02	+ .03	+ .05	+ .06	+ .08	+ .10	+ .12	290
80	.00	.00	.00	+ .01	+ .01	+ .02	+ .02	+ .03	+ .04	+ .05	+ .06	280
90	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	270
100	.00	.00	.00	− .01	− .01	− .02	− .02	− .03	− .04	− .05	− .06	260
110	.00	.00	− .01	− .01	− .02	− .03	− .05	− .06	− .08	− .10	− .12	250
120	.00	.00	− .01	− .02	− .03	− .05	− .07	− .09	− .12	− .15	− .18	240
130	.00	.00	− .01	− .02	− .04	− .06	− .09	− .12	− .16	− .19	− .23	230
140	.00	.00	− .01	− .03	− .05	− .08	− .11	− .15	− .19	− .23	− .28	220
150	.00	.00	− .01	− .03	− .06	− .09	− .12	− .16	− .21	− .26	− .31	210
160	.00	.00	− .02	− .03	− .06	− .09	− .13	− .18	− .23	− .28	− .34	200
170	.00	.00	− .02	− .04	− .06	− .10	− .14	− .19	− .24	− .30	− .36	190
180	0.00	0.00	−0.02	−0.04	−0.06	−0.10	−0.14	−0.19	−0.24	−0.30	−0.36	180

We see from (3) that if the spectrograms have all been exposed for the same length of time the correction is proportionate to the true velocity at the corresponding mean phase; consequently the effect of the error that we are discussing would be to diminish the semi-amplitude of oscillation without changing in any other way the shape of the velocity-curve. While this is strictly true only if the lengths of exposures are the same, it will be approximately true in any case that will arise in practice, so long as the average length of exposure is about the same for various parts of the velocity-curve.

A convenient procedure for applying this correction is as follows: The observations are plotted in the usual way and from this is derived a first approximation, K', to the semi-amplitude. This will be too small by an amount that we may ascertain from the first line in the table. Thus, if the average length of exposure is four-tenths of the period, we find 0.24 below this in the first line, and therefore a better approximation, K'', to the semi-amplitude may be found thus:

$$K''=K'\,\frac{1}{1-0.24}=1.32\,K'.$$

We now apply to each measured velocity the product of K'' and the corresponding number in the table.

If we develop (3) in ascending powers of u_2-u_1 we obtain

$$K\cos\frac{u_2+u_1}{2}\left\{\frac{(u_2-u_1)^2}{24}-\frac{(u_2-u_1)^4}{1920}+\text{etc.}\right\}. \qquad (4)$$

From this we see that whatever the mean phase may be, the correction depending upon the length of exposure is very nearly proportionate to the square of the length of exposure. Incidentally we see that

$$K\left(\frac{u_2-u_1}{5}\right)^2\cos\frac{u_2+u_1}{2}$$

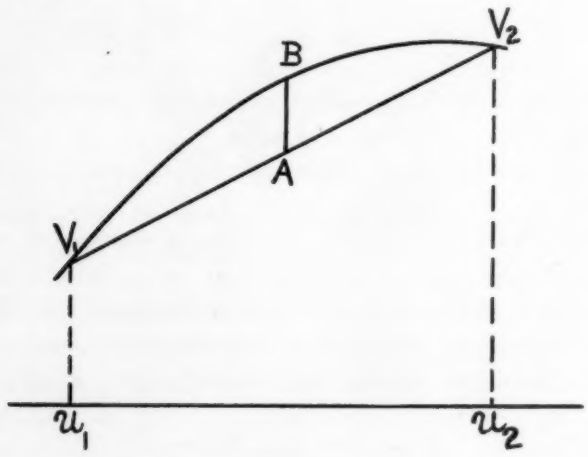
is a good approximation to the correction even for very large values of u_2-u_1.

An even better approximation is obtained by subtracting from the true velocity the mean of the velocities at u_1 and u_2; this difference is very closely equal to three times the correction, the error being always less than

$$K\frac{(u_2-u_1)^4}{2880}\cos\frac{u_2+u_1}{2}.$$

This approximation lends itself readily to graphical evaluation. In the diagram let V_1V_2 be a portion of the true velocity-curve, V_1 corresponding to u_1, and V_2 to u_2. From the middle point of the chord V_1V_2 erect the vertical AB; the correction is equal to one-third of AB.

We have confined our attention to circular orbits. There would be no particular difficulty in extending the present results to include elliptic orbits, but the necessity for this has not yet arisen, and there is reason to believe that it cannot arise, the orbits of all short-period binaries being very nearly circular. It is worthy of remark that the graphical correction described in the preceding paragraph would apply nearly as well to orbits of moderate eccentricity as to circular orbits.



Long exposures not only cause the systematic error that we have discussed, but they also tend to increase accidental error in a number of ways: (1) changes of temperature in the spectrograph are likely to be greater; (2) differential flexure is greater; (3) the effective mean time of exposure may be uncertain by reason of changes in the transparency of the atmosphere and in the seeing; (4) if the spectrogram is exposed near the times of maximum or minimum velocity, the lines may be made asymmetric, and the observer may have considerable difficulty in deciding upon what portion of the line to make the micrometer settings; this error would probably vary to considerable extent with the strength of the spectrogram and with the manner in which it was developed.

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NOTE ON "TERRESTRIAL" RADIATION

The radiation from the surface of the earth which does not escape immediately to space is absorbed, mainly by the aqueous vapor in an air layer close to the surface, and is thereafter passed on from layer to layer as air radiation in a vertical progressive propagation outward to space, but a slower one than that of the immediately transmitted part.

We cannot expose an instrument having a horizontal surface of indefinitely great extent, and thus resembling the earth's surface, and allow such an instrument to radiate out to space through the atmosphere; but by using a small instrument whose radiating surface constitutes virtually an element of the desired surface, we can let it radiate to a correspondingly small element of sky close to the zenith with identically the same relative radiating power that the sum of all the elements in the widely extended surface of the earth exerts to the similarly extensive sum of the sky elements above that surface. Or what is the same thing, the rate of transmission of radiant energy per square centimeter of surface between each square kilometer of the earth's surface and a parallel layer of atmosphere a few hundred meters thick immediately above the surface (which is practically the absorbent layer) is identically that of an instrument of 1 sq. cm in a vertical direction through an air column of this section.

As a convenient mode of measuring radiation to the sky, the instrument may have a wider angular aperture than the ideal one, but the principle remains the same. A curve of sky radiation at different altitudes above the horizon, obtained with an instrument whose angular aperture covered 24 sq. degrees of sky, has been published by me in the *American Journal of Science* for April, 1913 (Fig. 2, **35**, 383). A less accurate curve, obtained with an instrument of 32° angular aperture, or one which has more than thirty times the angular area of my instrument, and is therefore rather too coarse for such work, has been given by Mr. Anders Ångström in the *Astrophysical Journal* for January, 1914 (Fig. 2, **39**, 97). These curves may be summed hemispherically by zones, as Mr. Ångström has done, and if the experiment is performed by night the sum should be equal to the radiation of the same instrument

exposed horizontally to the entire hemisphere of the sky, and may indeed be labeled "nocturnal radiation"; but this is a purely instrumental or laboratory fact, and represents no fact of nature. The *terrestrial* radiation to space, which is correctly given by the process described above, is more than twice as great; and therefore where Ångström gets transmissions of 5 to 25 per cent for "nocturnal radiation," I get an average of 42 per cent for the transmission of terrestrial radiation in middle latitudes. Dr. Nils Ekholm[1] gets results in Scandinavia in entire agreement with mine, obtaining transmissions of 55 to 70 per cent, appropriate to a more northern clime.

Finally, I have found quite similar transmission (48 per cent) for radiation somewhat resembling that of the earth in quality, but coming in the opposite direction, namely, for that emitted by the heated surface of the moon, whence it follows that Ångström's values are far too low.

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[1] "Om Naturens Värmehushållning," *K. Vetenskapsakademiens Årsbok*, 1914, p. 314.